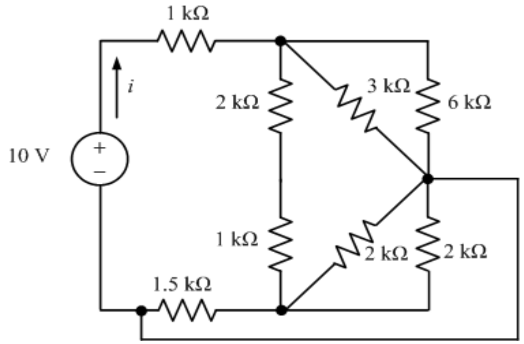
ELEC 2400 Electronic Circuits

Chapter 2: Resistive Networks and DC Analysis



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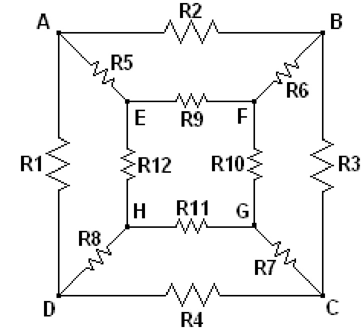
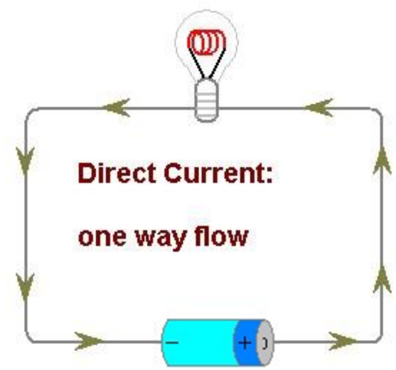
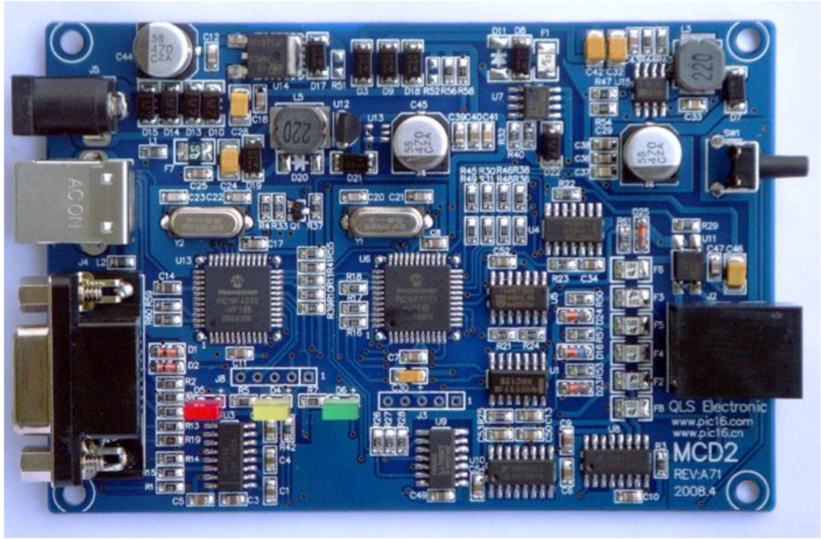
HKUST, 2021-22 Fall

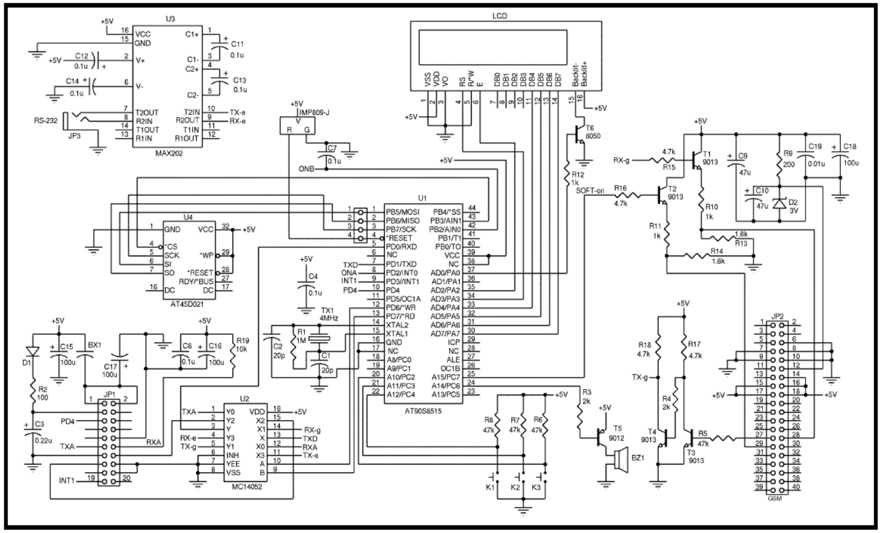
Chapter 2: Resistive Networks and DC Analysis





Circuit Hardware and Circuit Diagram





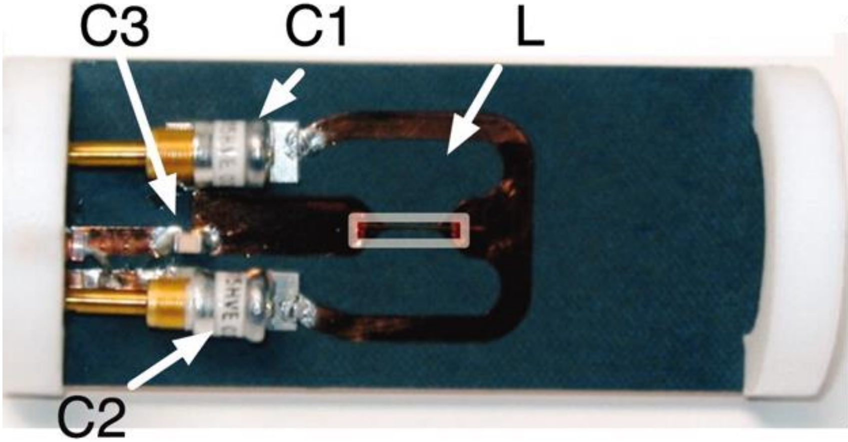
2.1 Circuit Terminology

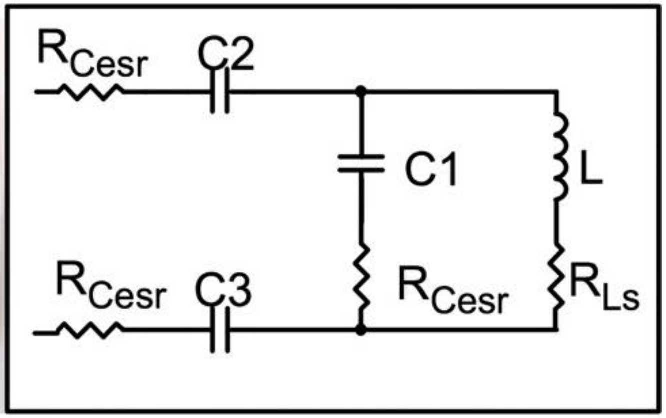
In this chapter, we will discuss fundamental laws of circuit theory. Some definitions of graph are introduced first.

Lumped parameter circuit:

(1) a circuit with physical dimensions small compared to the signal wavelength.

(2) the circuit is modeled as an interconnection of concentrated elements (resistors, capacitors, and inductors, etc.) joined by a network of perfectly conducting wires. The circuit elements have idealized lumped parameters (resistance, capacitance, inductance, etc.).





Circuit Terminology

Node: an electrical joint connecting the terminals of two or more circuit elements.

Branch: consists of two nodes between which a circuit elements is inserted.

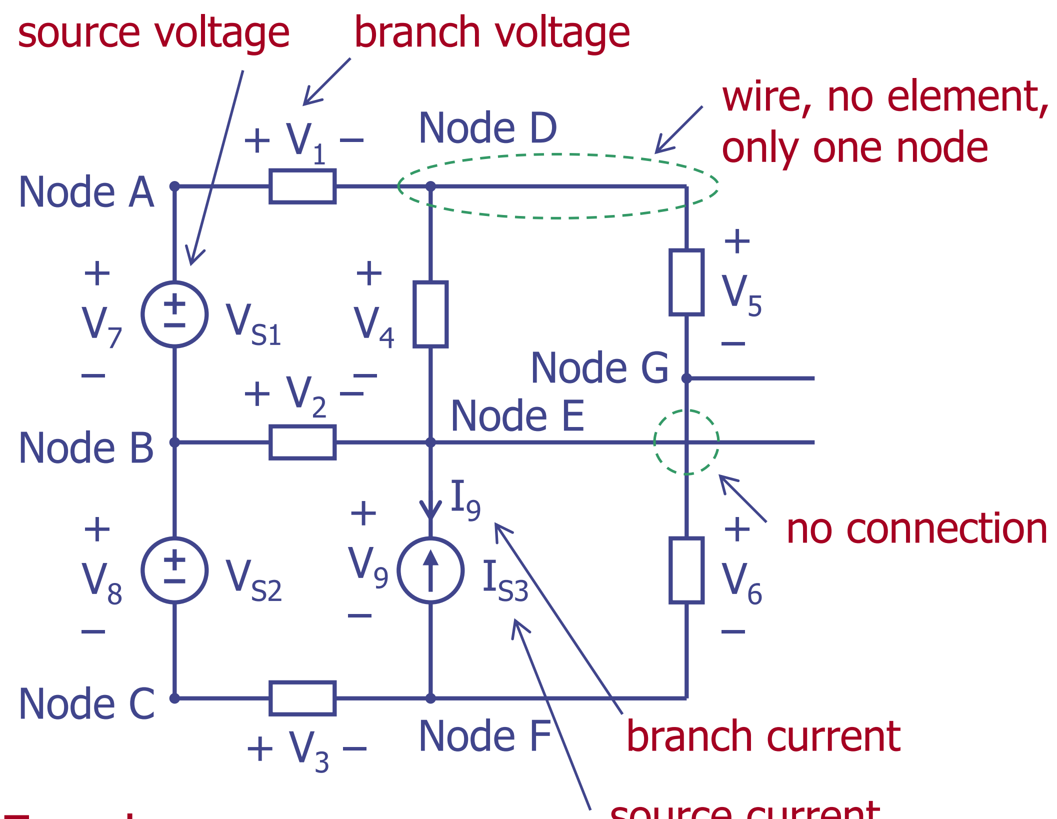
Path: a sequence of nodes proceeding from the starting node to the ending node.

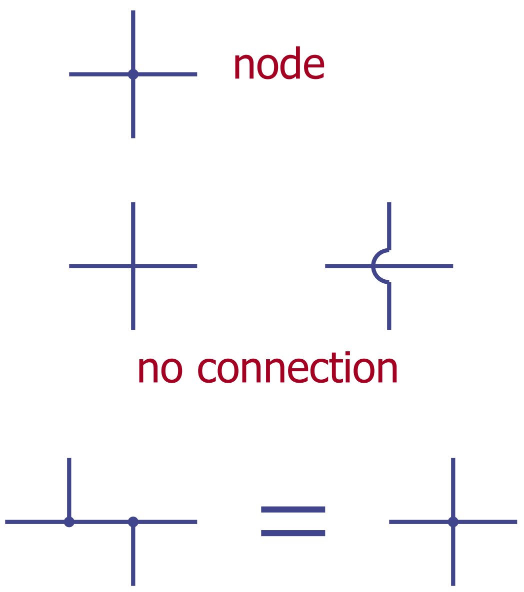
Loop: a closed path with the starting node the same as the ending node without passing an intermediate node more than once.

Mesh: a loop that does not contain any other loops within it.

Circuit Diagram

Circuit diagram: a graphical representation of a circuit (closed connections of circuit elements).



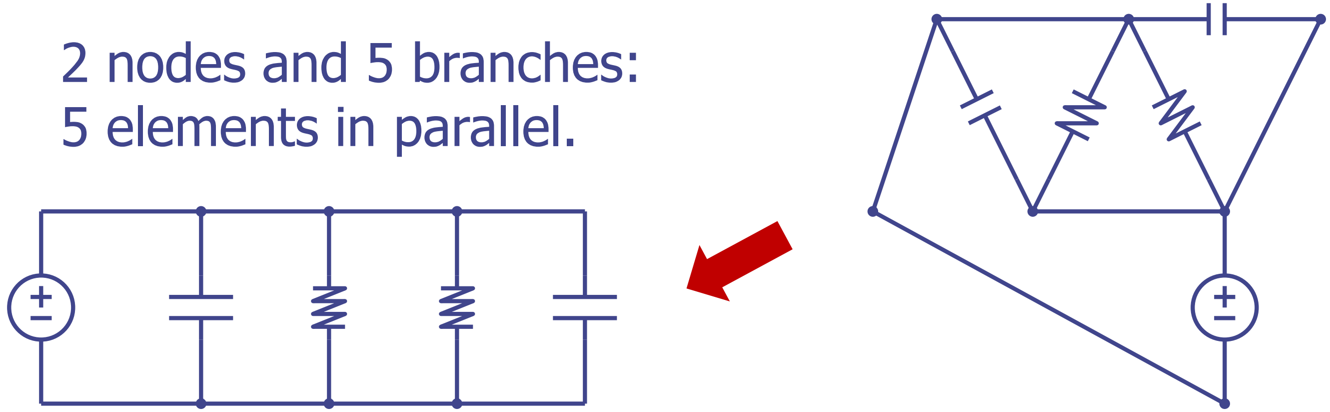


7 nodes
9 branches

Examples 2-1, 2-2

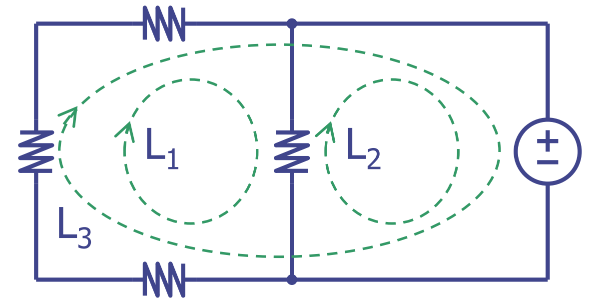
Example 2-1: How many nodes and branches are there in the circuit diagram on the right?

Soln: 2 nodes and 5 branches:
5 elements in parallel.



Example 2-2: How many loops and meshes are there in the circuit diagram?

Soln: 3 loops and 2 meshes.



Chapter 2: Resistive Networks and DC Analysis

2.1 Circuit Terminology

2.2 Circuit Laws

- **2.2.1 Kirchhoff's Current Law**
- **2.2.2 Kirchhoff's Voltage Law**

2.3 Resistive Network

- 2.3.1 Resistors in Series and in Parallel
- 2.3.2 Voltage and Current Dividers

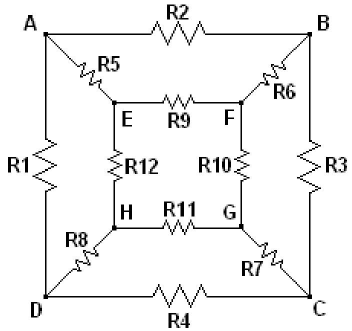
2.4 Circuit Analysis

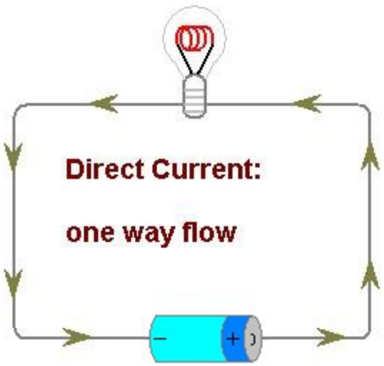
- 2.4.1 Nodal Analysis
- 2.4.2 Loop and Mesh Analysis
- 2.4.3 Superposition

2.5 Maximum Power Transfer & High-Voltage Transmission

2.6 Equivalence and Source Transformation

- 2.6.1 Thevenin's and Norton's Theorems
- 2.6.2 General Proof



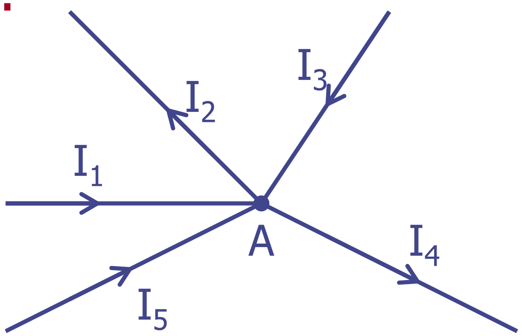


2.2.1 Kirchhoff's Current Law

Kirchhoff's Current Law (KCL):

At any instant of time, the algebraic sum of the currents entering (or leaving) a node of a circuit is equal to 0, i.e., $\Sigma I_i = 0$.

Example 2-3:



Gustav Robert Kirchhoff
(1824-1887)

Consider currents entering Node A:

$\Rightarrow \quad I_1 - I_2 + I_3 - I_4 + I_5 = 0$

N.B. I_2 and I_4 bear minus signs because they are leaving A.

Current In = Current Out

KCL is a consequence of conservation of charge: charge entering a node must leave that node instantaneously, and there is no charge accumulation at the node.

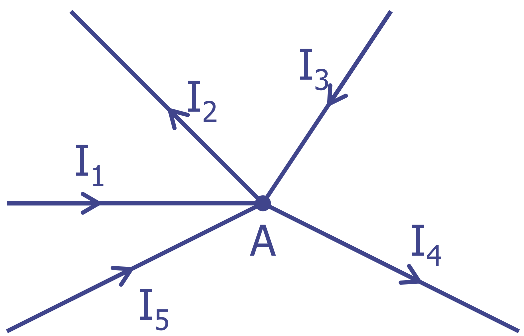
$$\Sigma q_i = 0 \;\Rightarrow\; \Sigma \frac{dq_i}{dt} = 0 \;\Rightarrow\; \Sigma I_i = 0$$

KCL is more conveniently stated as:

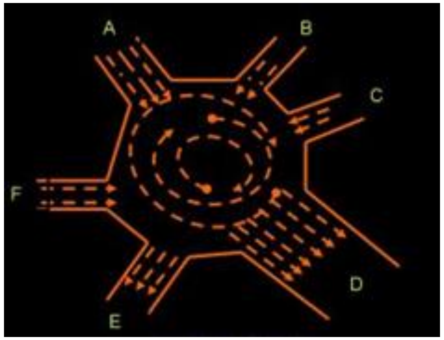
$$\Sigma I_i = \Sigma I_k$$

where I_j's and I_k's are currents entering and leaving the node, respectively.

Example 2-4:

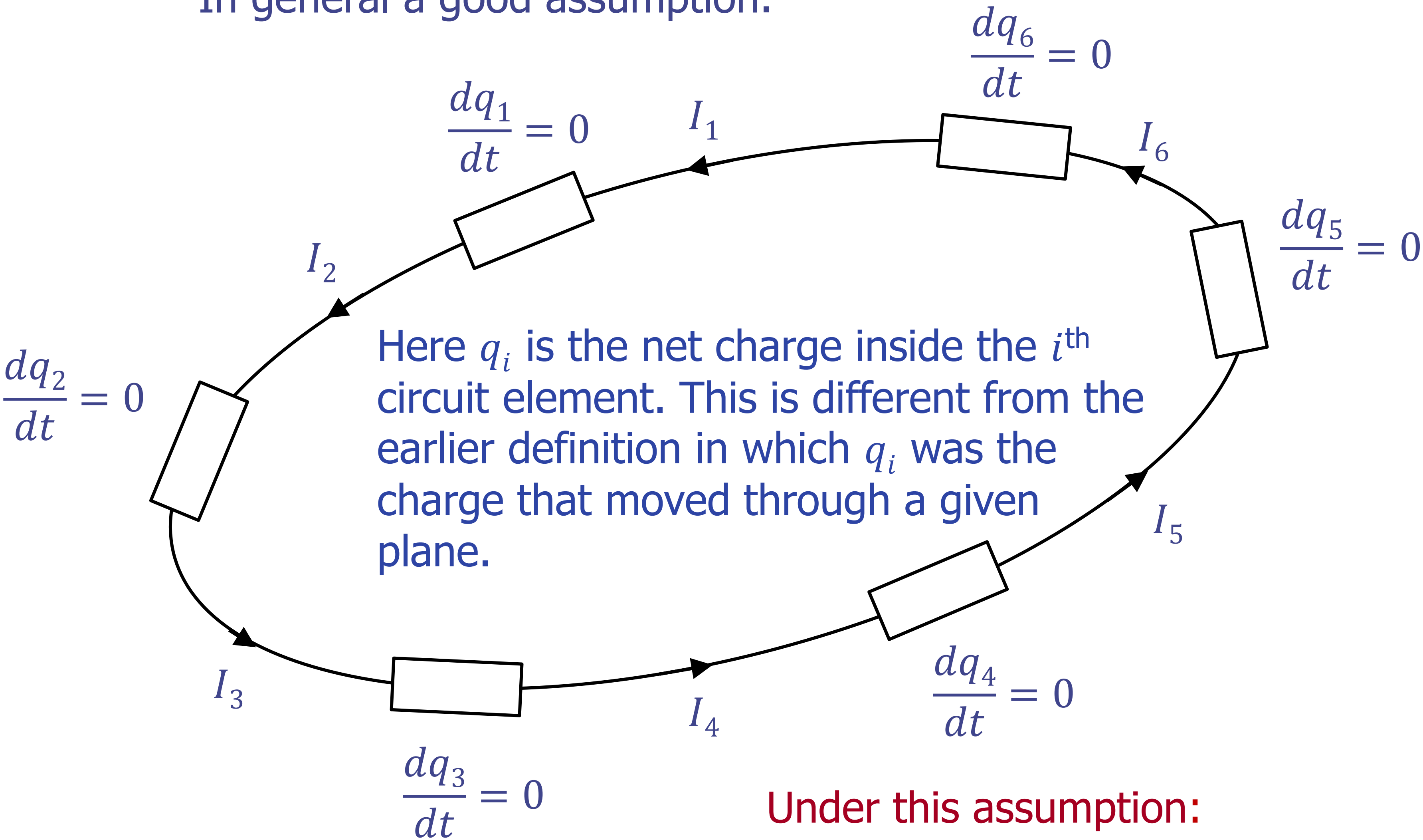


$$I_1 + I_3 + I_5 = I_2 + I_4$$



Additional Assumption for KCL

No change of net charge within any circuit elements.
In general a good assumption.



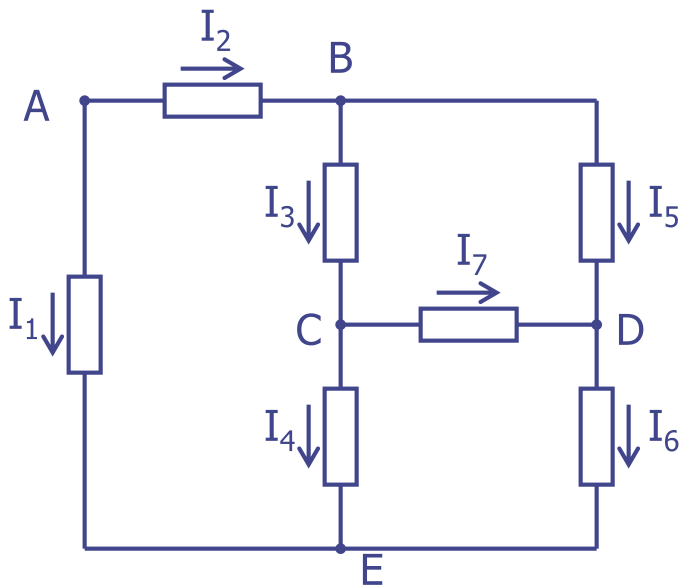
Here q_i is the net charge inside the i^{th} circuit element. This is different from the earlier definition in which q_i was the charge that moved through a given plane.

Under this assumption:

$I_1 = I_2 = I_3 = I_4 = I_5 = I_6$

Examples 2-5, 2-6

Example 2-5:



Node A: $I_1 + I_2 = 0$

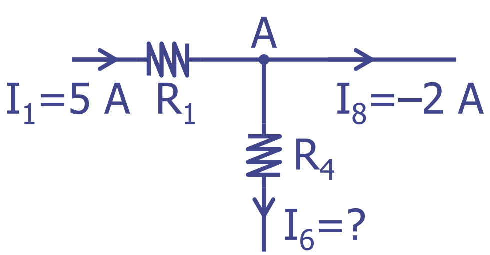
Node B: $I_2 = I_3 + I_5$

Node C: $I_3 = I_4 + I_7$

Node D: $I_5 + I_7 = I_6$

Node E: $I_1 + I_4 + I_6 = 0$

Example 2-6: Find I_6.



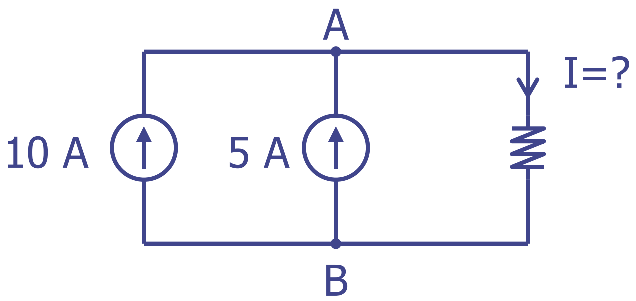
Soln.:

$$I_1 = I_6 + I_8$$

$$\Rightarrow \quad I_6 = 7\ A$$

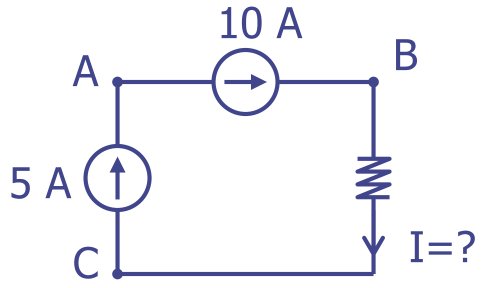
Examples 2-7, 2-8, 2-9

Example 2-7: Find I.



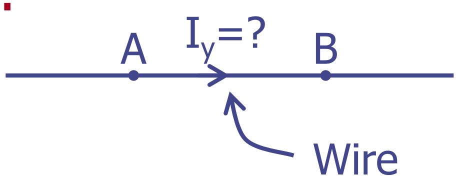
Soln.: I = 15 A

Example 2-8: Find I.



Soln.:
Invalid circuit because KCL is violated at Node A (5 A $\neq$ 10 A).

Example 2-9:



Is Node A = Node B?
Is $I_y = 0$?
Is $V_{AB} = 0$?

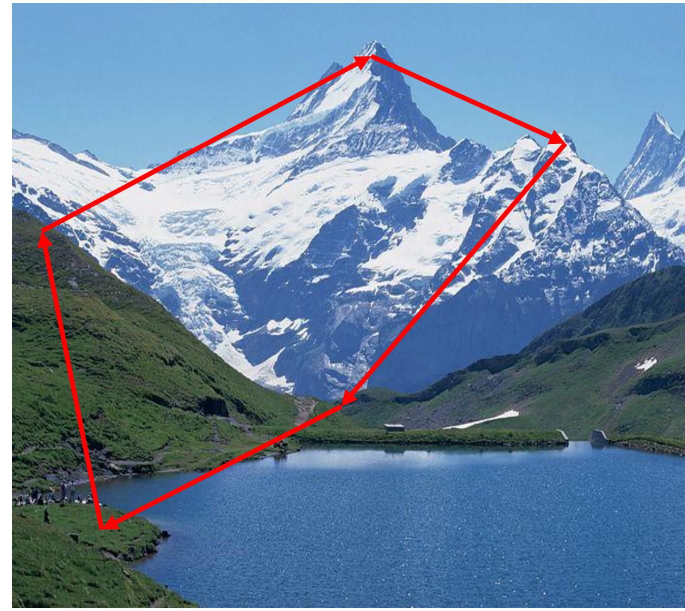
Soln.:

Node A = Node B.
I_y cannot be determined here.
$V_{AB} = 0$.

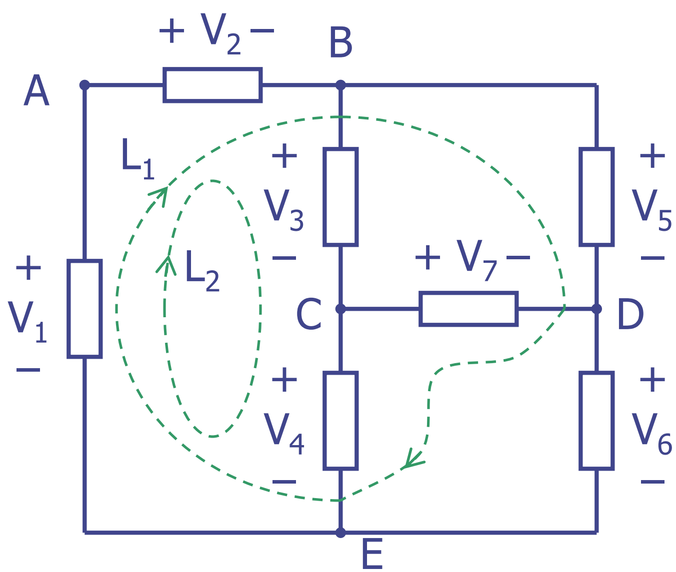
2.2.2 Kirchhoff's Voltage Law

Kirchhoff's Voltage Law (KVL):

At any instant of time, the algebraic sum of the branch voltages around a loop of a circuit is equal to zero, i.e., $\Sigma V_i = 0$.



Example 2-10:



Loop 1 (L_1, voltage rise is positive, voltage drop is negative):

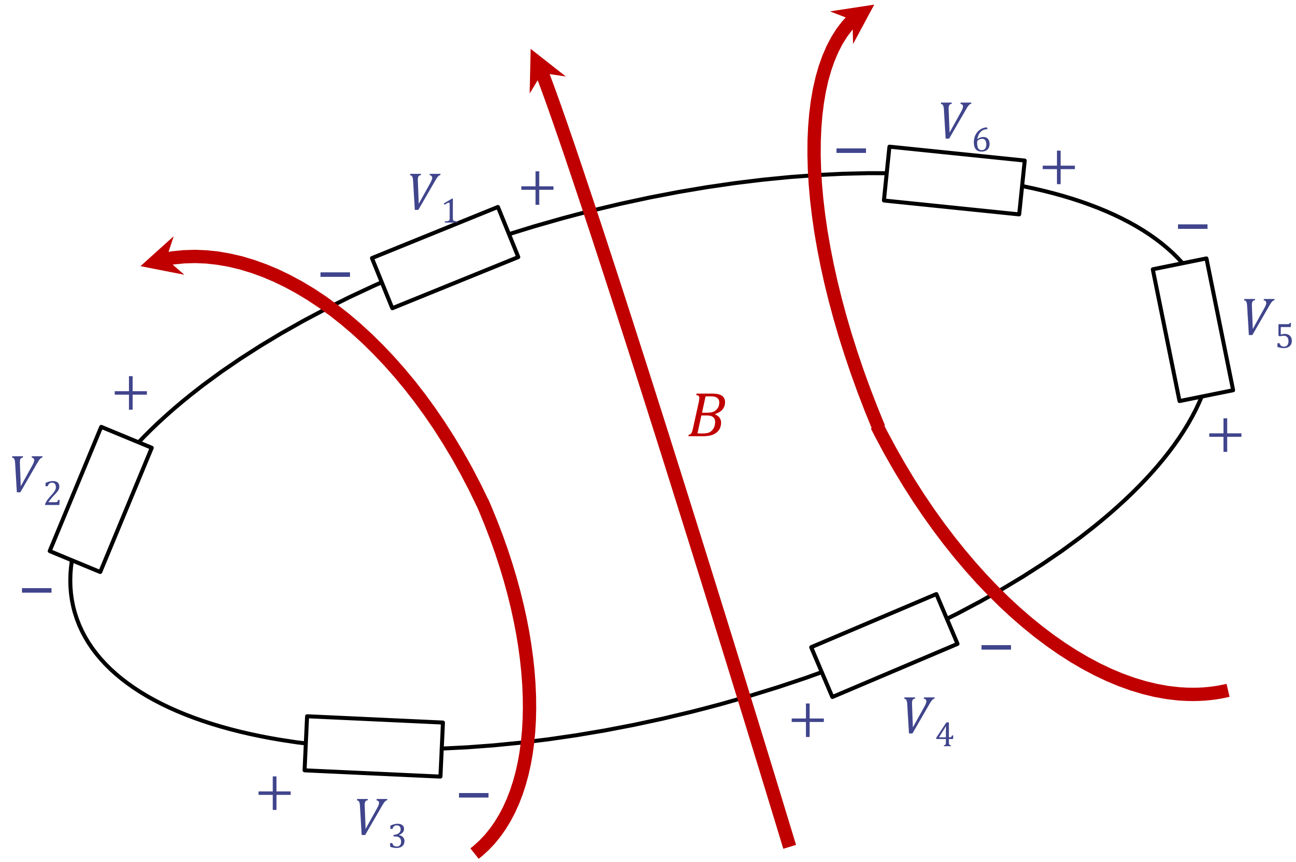
$$+V_1 - V_2 - V_5 + V_7 - V_4 = 0$$

Loop 2 (L_2):

$$+V_1 - V_2 - V_3 - V_4 = 0$$

Main Assumption for KVL

No change of magnetic field outside circuit elements



$\frac{dB}{dt} = 0 \Rightarrow$ No electromagnetic induction.

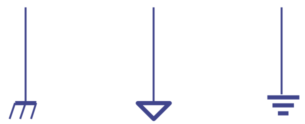
Electric field is conservative and hence $\Sigma V_i = 0$.

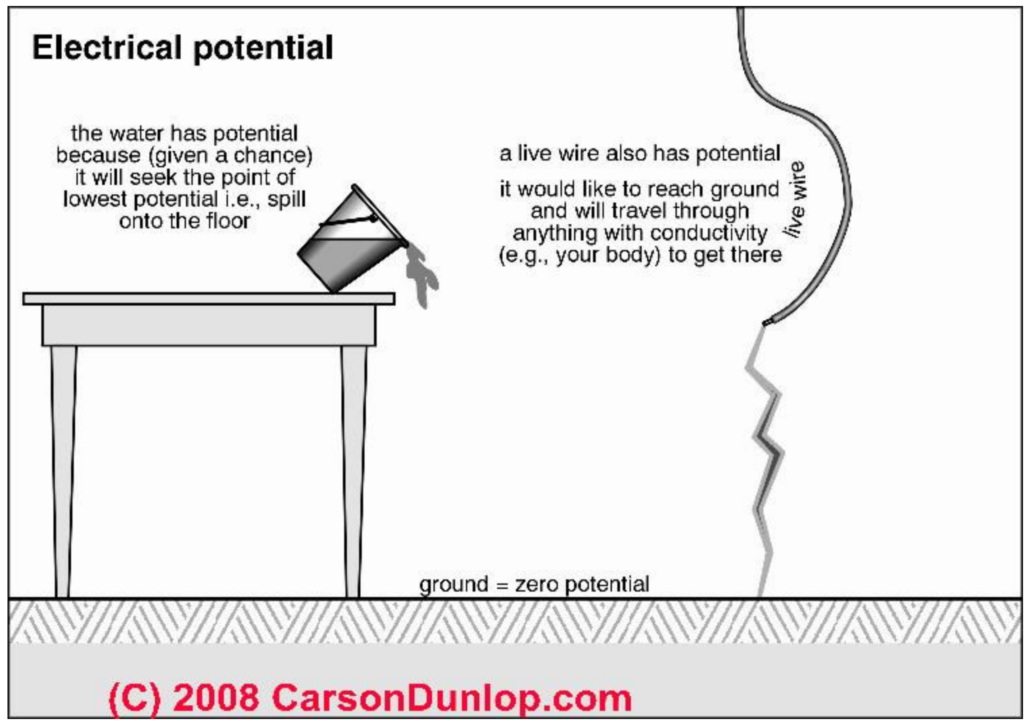
Conservation of Energy and Ground Potential

KVL is a consequence of conservation of energy, with voltage being energy per unit charge ($V = E/q$). An increase in energy from A to B is thus identified as a rise in voltage, while a decrease in energy is a drop in voltage.

Voltage is a relative quantity, and it is convenient to specify a reference node for the whole circuit, usually known as the ground node, or simply ground (GND), assigned as 0 V.

Symbols for Ground



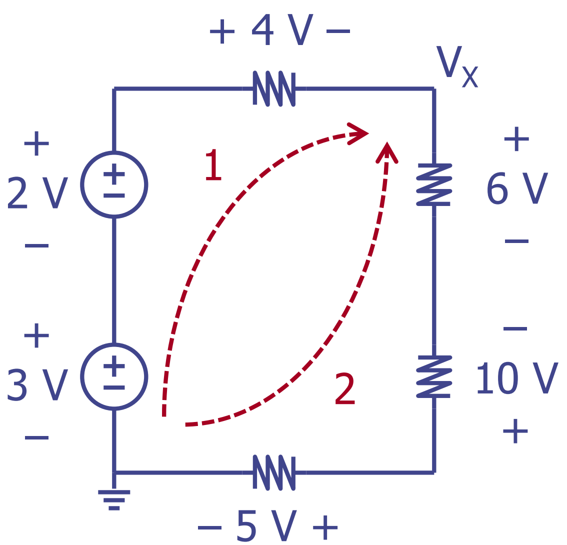


Equivalent Statement of KVL

With the introduction of the ground node, KVL can be restated:

> The voltage at a node, with reference to ground, is the algebraic sum of the branch voltages that constitute a path from ground to that node.

Example 2-11: Find V_x.



Soln.:
Consider the two paths from ground to V_x.

Path 1:

$$V_x = +3\text{ V} + 2\text{ V} - 4\text{ V} = 1\text{ V}$$

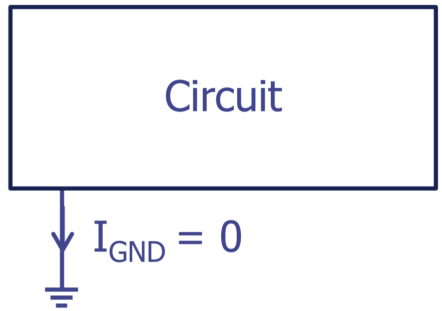
Path 2:

$$V_x = +5\text{ V} - 10\text{ V} + 6\text{ V} = 1\text{ V}$$

External Ground and Supply Connections

(A) Single Ground Connection. No Other External Connections:

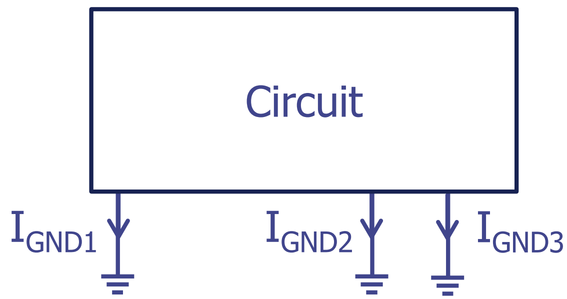
$I_{GND} = 0$ because ground is a dead end and there is no return path.



(B) Multiple Ground Connections:

I_{GND}'s are not necessarily zero, but total ground current is zero, i.e.,

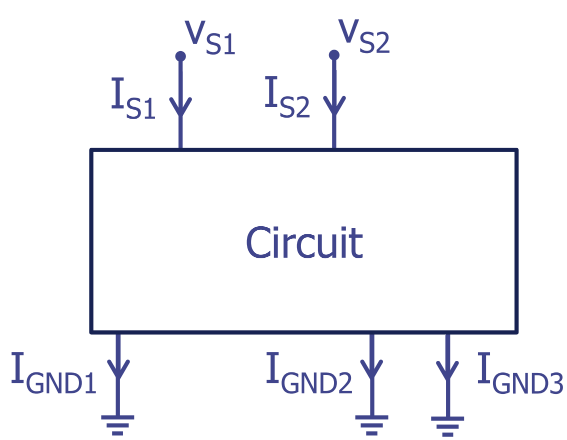
$I_{GND1} + I_{GND2} + I_{GND3} = 0$



(C) Multiple Ground and Supply Connections:

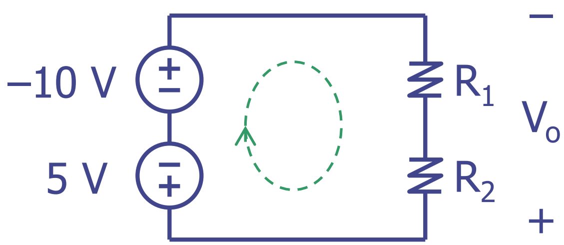
Total supply current is equal to total ground current, i.e.,

$I_{S1} + I_{S2} = I_{GND1} + I_{GND2} + I_{GND3}$



Examples 2-12, 2-13

Example 2-12: Find V_o.



Soln.:

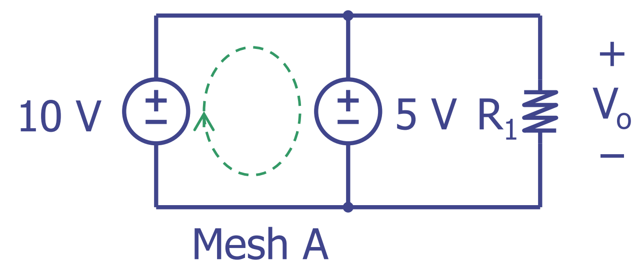
Apply KVL to the mesh:

$$-5\text{ V} - 10\text{ V} + V_o = 0\text{ V}$$

$$V_o = 15\text{ V}$$

Be careful about the signs!

Example 2-13: Find V_o.



Soln.:

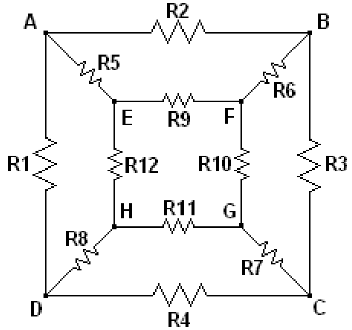
KVL of Mesh A should give $\Sigma V_i = 0$, but

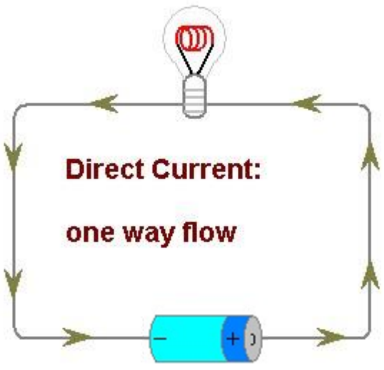
$$10\text{ V} - 5\text{ V} = 5\text{ V} \neq 0\text{ V}$$

and no solution exists (invalid circuit).

Remark: Voltage sources of unequal values cannot be connected in parallel.

Chapter 2: Resistive Networks and DC Analysis



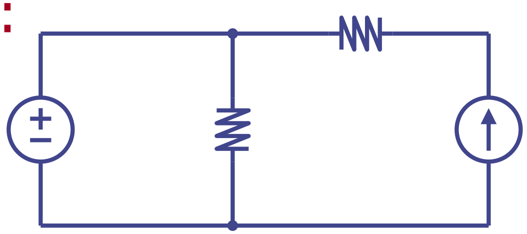


2.3 Resistive Network

Resistive network: consists of only resistors, voltage sources and current sources.

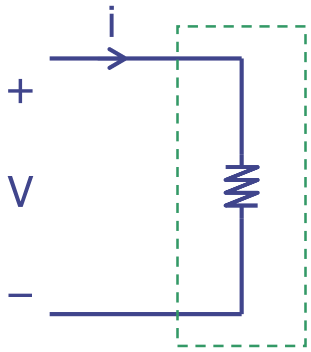
N. B. A network is a more complicated circuit; and network is usually interchangeable with circuit.

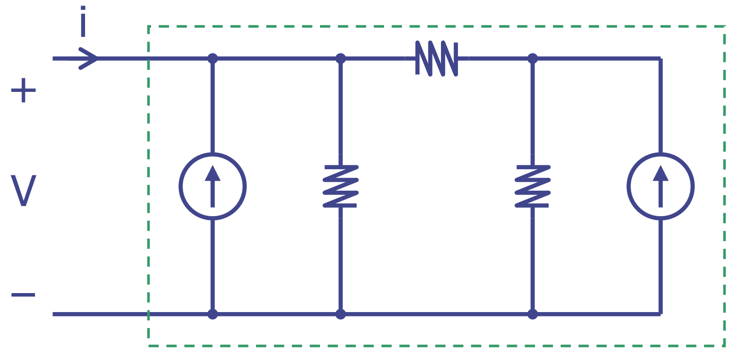
Example 2-14:



3 nodes, 4 elements

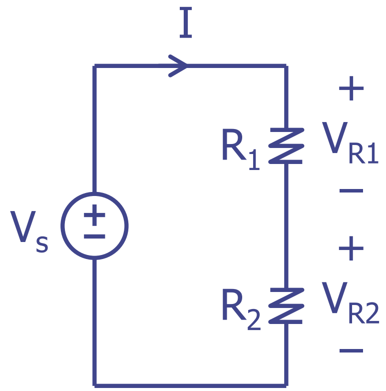
Examples of resistive 1-port network:





2.3.1 Resistors in Series

Consider connecting two resistors in series:



KCL mandates

$$I_{R1} = I_{R2} = I$$

KVL gives

$$V_s = V_{R1} + V_{R2}$$
$$= I\times R_1 + I\times R_2 \quad \text{(Ohm's law)}$$
$$= I\times(R_1+R_2)$$

Hence, the equivalent resistance R_{eq} is

$$R_{eq} = R_1 + R_2$$

In general, for n resistors connected in series:

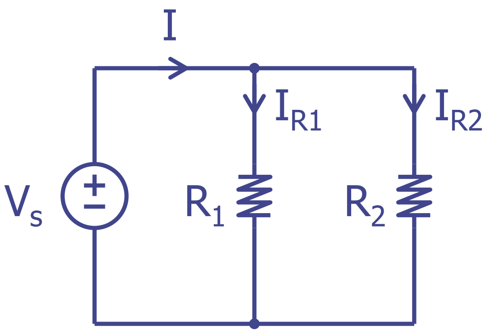
$$R_{eq} = R_1 + R_2 + \ldots + R_n$$
$$= \sum_{k=1}^{n} R_k$$

For resistors in Series: 1) The current is the same.

2) The order of connection is immaterial.

Resistors in Parallel

Consider connecting two resistors in parallel:



KVL mandates that

$$V_s = V_{R1} = V_{R2}$$

KCL gives

$$I = I_{R1} + I_{R2}$$

$$= \frac{V_s}{R_1} + \frac{V_s}{R_2} \qquad \text{(Ohm's law)}$$

$$= V_s\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

In general, for n resistors connected in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n} = \sum_{k=1}^{n} \frac{1}{R_k}$$

For resistors in parallel: 1) The voltage is the same.

2) The order of connection is immaterial.

2 and 3 Resistors in Parallel

For R_1 and R_2 connected in parallel, R_{eq} is written as $R_{eq} = R_1||R_2$, and

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\Rightarrow \quad R_{eq} = R_1 || R_2 = \frac{R_1 R_2}{R_1 + R_2} \left(= \frac{\text{product}}{\text{sum}} \right)$$

For 3 resistors in parallel, apply the formula twice and

$$R_{eq} = R_1 || R_2 || R_3 = \frac{R_1 R_2}{R_1 + R_2} || R_3 = \frac{\frac{R_1 R_2}{R_1 + R_2} R_3}{\frac{R_1 R_2}{R_1 + R_2} + R_3}$$

$$= \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

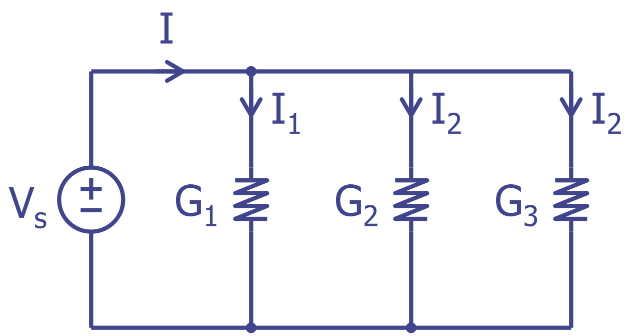
(Too complicated to be useful!)

Note that $R_1||R_2||R_3 \neq \cancel{\dfrac{R_1 R_2 R_3}{R_1 + R_2 + R_3}}$ (Wrong dimension!)

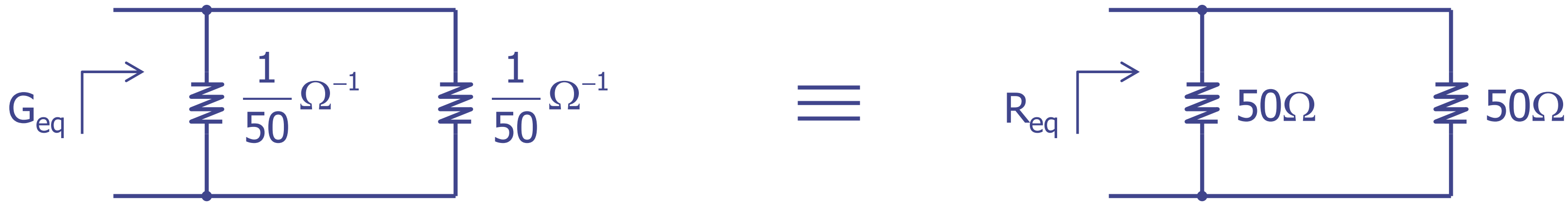
Resistance and Conductance

In dealing with resistors connected in parallel, conductance G (= 1/R) may be used to simplify analysis:

$$
\begin{aligned}
I &= I_1 + I_2 + I_3 \\
&= G_1 V_s + G_2 V_s + G_3 V_s \\
&= V_s (G_1 + G_2 + G_3) \\
G_{eq} &= G_1 + G_2 + G_3
\end{aligned}
$$



Example 2-15: Find G_{eq} and R_{eq}.

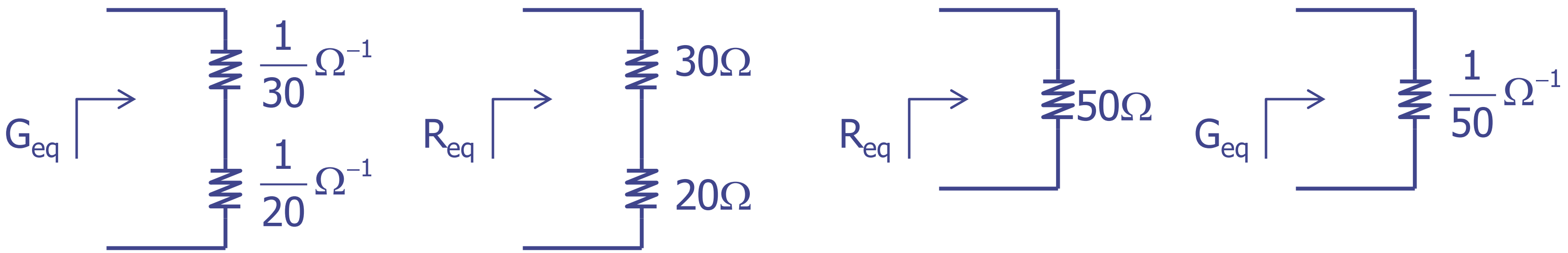


G_{eq} = 0.02 + 0.02
= 0.04 Ω^{-1} (or 0.04 S, S = Siemens)

R_{eq} = 50||50
= 25 Ω

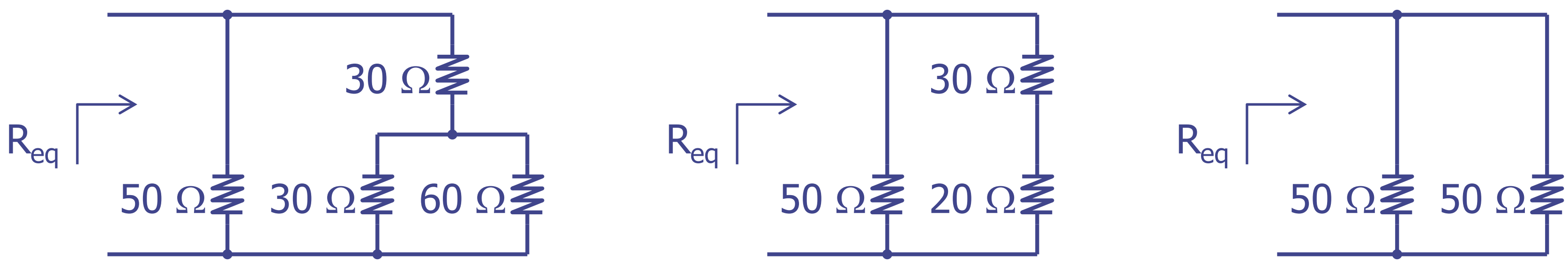
Examples 2-16, 2-17

Example 2-16: Find G_{eq}.



$G_{eq} = 0.02$ S

Example 2-17: Find R_{eq}.



$$30\Omega \,||\, 60\Omega = \frac{30 \times 60}{30 + 60} = 20\Omega$$

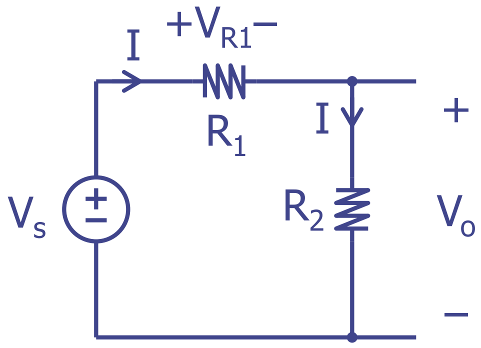
$$30\Omega + 20\Omega = 50\Omega$$

$$R_{eq} = 50\Omega \,||\, 50\Omega = 25\Omega$$

2.3.2 Voltage Divider

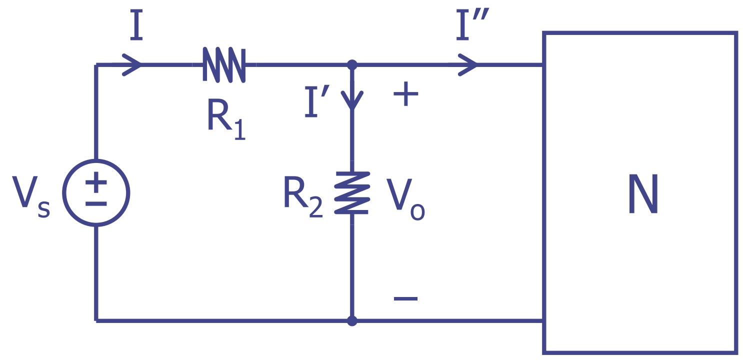
A voltage divider circuit can be formed from using two resistors:

$$V_s = V_{R1} + V_o$$
$$= I \times R_1 + I \times R_2$$
$$\frac{V_o}{V_s} = \frac{R_2}{R_1 + R_2}$$



By choosing appropriate R_1 and R_2, we can obtain any voltage V_o between 0 and V_s.

However, it should be noted that this V_o cannot be used to drive any load. For the circuit to the right, if the network N takes in a current I″, then $I' = I - I'' \neq I$, and the above V_o / V_s relation will not hold.



Current Divider

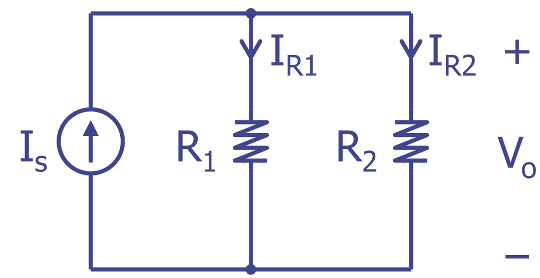
A current divider is the dual of the voltage divider:

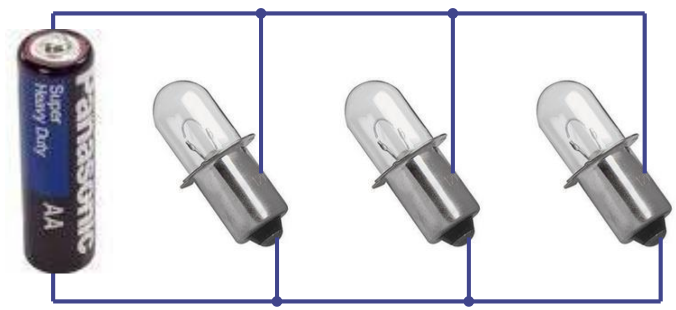
$$I_s = I_{R1} + I_{R2}$$

$$= \frac{V_o}{R_1} + \frac{V_o}{R_2} = \frac{V_o}{R_1 \,||\, R_2}$$

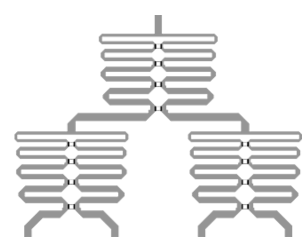
$$\frac{I_{R1}}{I_s} = \frac{V_o / R_1}{V_o / (R_1 \,||\, R_2)}$$

$$= \frac{R_2}{R_1 + R_2}$$



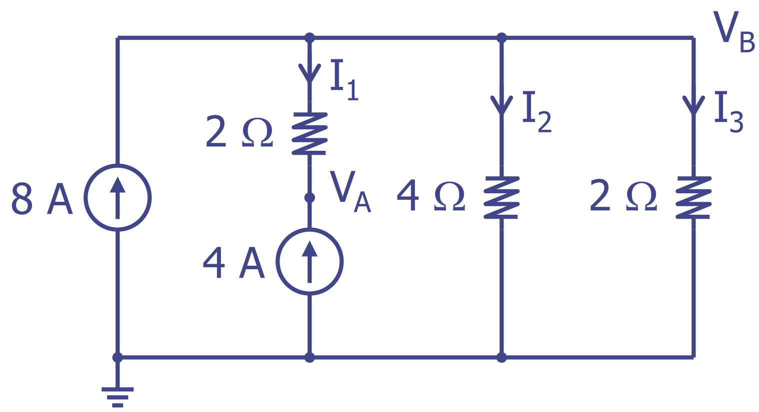


Note that I_{R1} is proportional to R_2, and I_{R2} is proportional to R_1. Also be reminded that the smaller resistor draws the larger current.



Example 2-18

Example 2-18: Find I_1, I_2, I_3, V_A and V_B.



Soln.:

$$I_1 = -4A$$

$$I_2 = (8A + 4A)\frac{2}{2+4} = 4A \quad \text{Current divider}$$

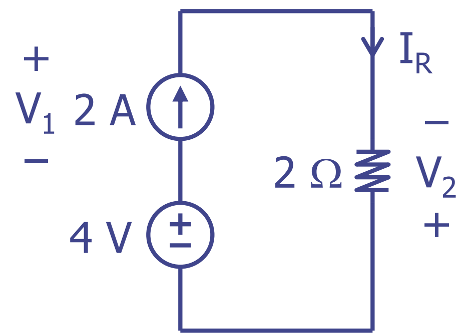
$$I_3 = 12A - I_2 = 8A$$

$$V_B = I_3 \times 2\Omega = 16V$$

$$V_A = V_B - (-4A \times 2\Omega) = 24V$$

Examples 2-19, 2-20

Example 2-19: Find I_R, V_1 and V_2.



Soln.:

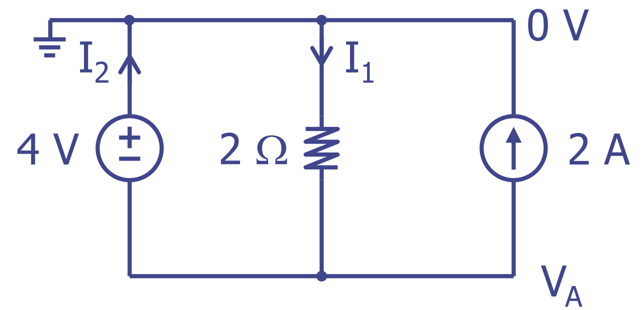
$I_R = 2$ A

$V_2 = -I_R \times 2\ \Omega = -4$ V

$4\text{ V} + V_1 = -V_2 = 4$ V

$V_1 = 0$ V !

Example 2-20: Find V_A, I_1 and I_2.



Soln.:

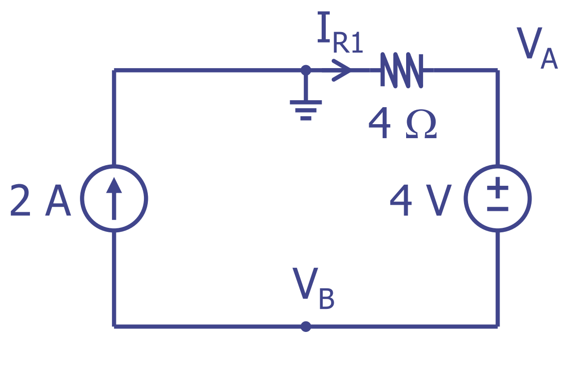
$I_1 = 4\text{ V}/2\ \Omega = 2$ A

$I_2 = I_1 - 2\text{ A} = 0$ A

$V_A = 0 - 4 = -4$ V

Examples 2-21, 2-22

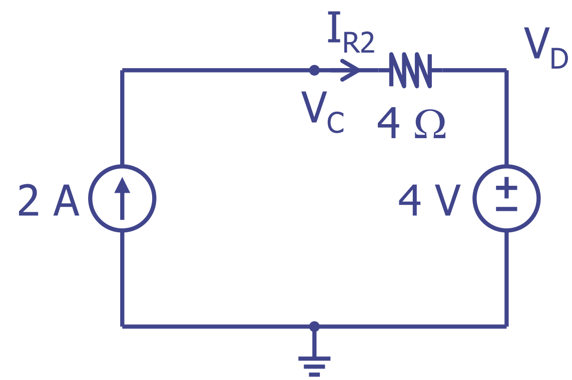
Example 2-21: Find I_{R1}, V_A and V_B.



Soln.:

I_{R1} = 2 A
V_A = 0 − 2×4 = −8 V
V_B = V_A − 4 = −8 − 4
= −12 V

Example 2-22: Find I_{R2}, V_C and V_D.



Soln.:

I_{R2} = 2 A
V_D = 0 + 4 = 4 V
V_C = V_D + 2×4 = 4 + 8
= 12 V

Chapter 2: Resistive Networks and DC Analysis

2.1 Circuit Terminology

2.2 Circuit Laws

- 2.2.1 Kirchhoff's Current Law
- 2.2.2 Kirchhoff's Voltage Law

2.3 Resistive Network

- 2.3.1 Resistors in Series and in Parallel
- 2.3.2 Voltage and Current Dividers

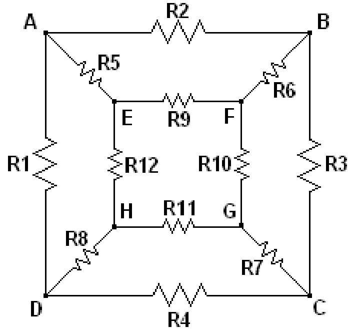
2.4 Circuit Analysis

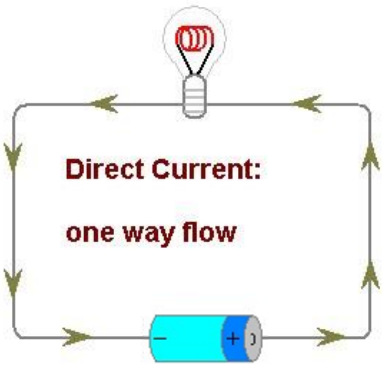
- **2.4.1 Nodal Analysis**
- **2.4.2 Loop and Mesh Analysis**
- **2.4.3 Superposition**

2.5 Maximum Power Transfer & High-Voltage Transmission

2.6 Equivalence and Source Transformation

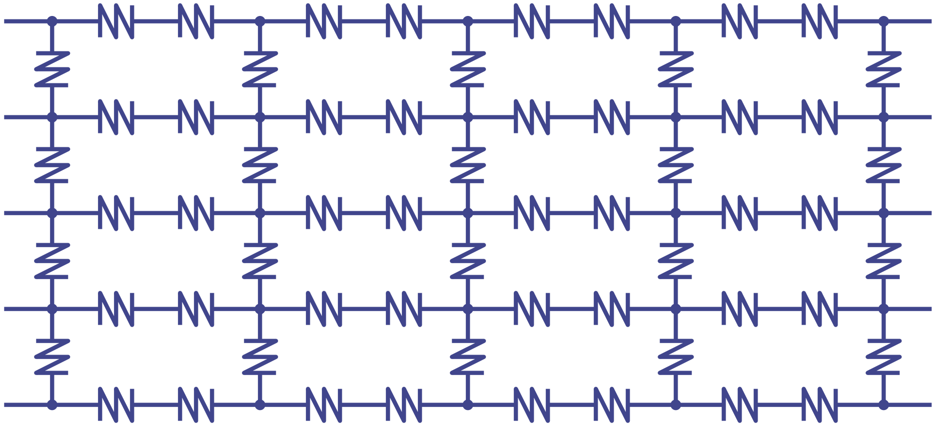
- 2.6.1 Thevenin's and Norton's Theorems
- 2.6.2 General Proof



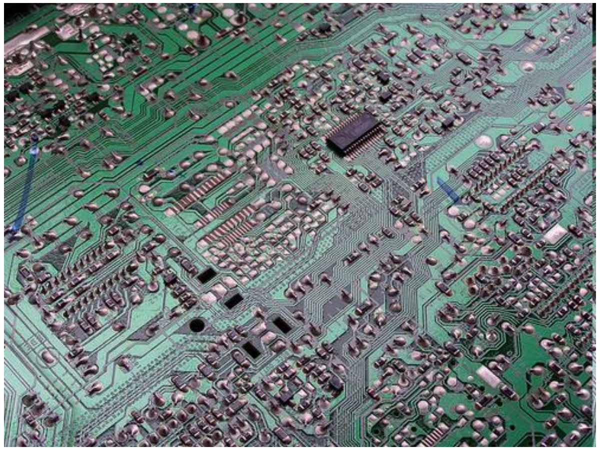


2.4.1 Nodal Analysis

A circuit can be very complicated, and one needs a systematic way to analyze it $\Rightarrow$ use nodal analysis.



rear windshield defrosting circuit



Procedure of Nodal Analysis

To solve a complicated circuit, nodal analysis (related to nodes) can be employed. The procedure is as following.

(1) For a circuit with n nodes, one node is assigned the ground (reference) node with node voltage of 0 V.

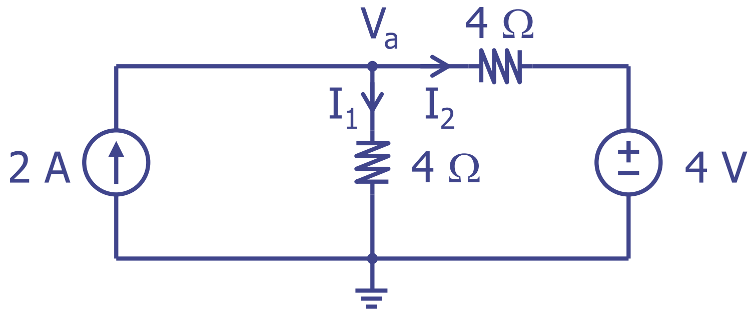
(2) Write (n–1) KCL equations at the (n–1) non-ground nodes. Alternatively, any (n–1) nodes can be chosen.

(3) For the (n–1) equations in (n–1) unknowns, we may solve them by the Gaussian elimination method.

After all node voltages are obtained, all branch currents can then be computed. In this course, we deal with at most 2 equations with 2 unknowns, and simple elimination method is adequate.

Example 2-23

Example 2-23 (circuit with voltage source): Solve for V_a and I_1.



Soln.: Write KCL equation at V_a.

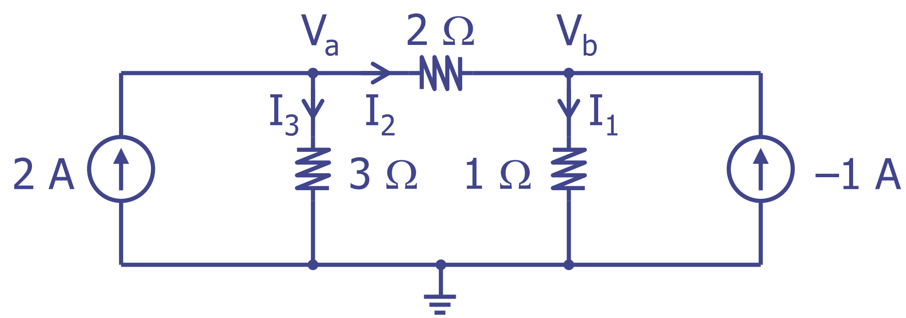
@V_a: $$2 = \frac{V_a}{4} + \frac{V_a - 4}{4}$$

$\Rightarrow$ $$2V_a - 4 = 8$$

Ans.: $$V_a = 6V, \quad I_1 = 1.5V$$

Example 2-24

Example 2-24: Solve for all node voltages and branch currents.



Soln.: Write KCL equations at V_a and V_b first.

@V_a: $$2 = \frac{V_a}{3} + \frac{V_a - V_b}{2} \Rightarrow 5V_a - 3V_b = 12 \quad (1)$$

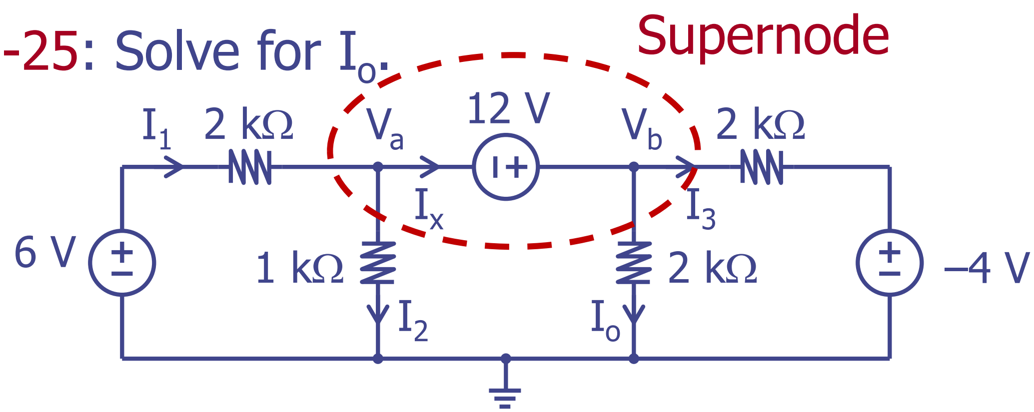
@V_b: $$\frac{V_a - V_b}{2} = \frac{V_b}{1} + 1 \Rightarrow V_a - 3V_b = 2 \quad (2)$$

(1) – (2): $4V_a = 10V$

Ans.: $$V_a = 2\frac{1}{2}V, \quad V_b = \frac{1}{6}V, \quad I_1 = \frac{1}{6}A, \quad I_2 = \frac{7}{6}A, \quad I_3 = \frac{5}{6}A.$$

Example 2-25

Example 2-25: Solve for I_o.



Soln.: The current I_x through the 12 V voltage source cannot be expressed in terms of V_a and V_b, but note that

$$I_1 = I_2 + I_x = I_2 + I_o + I_3$$

Hence, V_a and V_b can be "considered" as a supernode, and

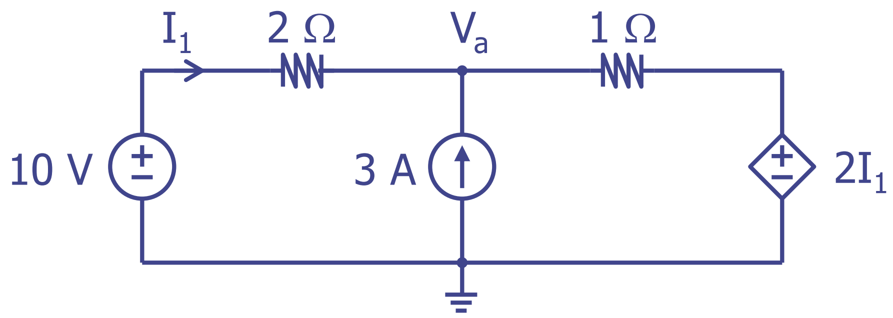
$$\frac{6 - V_a}{2k} = \frac{V_a}{1k} + \frac{V_a + 12}{2k} + \frac{V_a + 12 - (-4)}{2k}$$

$$\Rightarrow \qquad 6 - V_a = 2V_a + V_a + 12 + V_a + 16$$

$$\Rightarrow \qquad V_a = \frac{-22}{5}\,\text{V} \quad \text{and} \quad I_o = \frac{V_a + 12}{2k} = 3.8\text{mA}$$

Example 2-26

Example 2-26 (circuit with dependent source): Solve for V_a and I_1.



Soln.: Write KCL at Node A gives

$$I_1 + 3 = \frac{V_a - 2I_1}{1}$$

$$\Rightarrow \quad \frac{10 - V_a}{2} + 3 = V_a - 2\left(\frac{10 - V_a}{2}\right)$$

$$\Rightarrow \quad 10 - V_a + 6 = 4V_a - 20$$

Ans. $\quad V_a = 7.2V, \qquad I_1 = 1.4A$

SPICE Simulations

SPICE stands for Simulation Program with Integrated Circuit Emphasis.

SPICE was developed in 1973 at the University of California, Berkeley by Laurence Nagel and his research advisor, Prof. Donald Pederson. Nodal analysis is used in the simulations.

SPICE is a very important and powerful circuit-simulation program widely used by electrical engineers involved in circuit analysis and design. It can simulate electrical circuit behavior and calculate node voltages, branch currents, power, and other parameters of a circuit. An engineer can study the behavior of circuits without having to actually build them. The circuit can consist of resistors, capacitors, inductors, operational amplifiers, diodes, transistors, semiconductor devices, and other components.

PSPICE is a commercial version of SPICE from Cadence Design Systems. You will use it in your labs.

https://en.wikipedia.org/wiki/SPICE

2.4.2 Mesh Analysis (Optional)

If a circuit with n nodes and b branches can be laid out on a plane surface with no crossing of branches (planar graph), then it can be shown that there exist sets of b–(n–1) independent KVL equations related to loop/mesh currents, and mesh analysis can be used. (Loops and meshes are treated the same way.)

(1) Assign loops/meshes and the corresponding loop/mesh currents.

(2) For an element R_i that belongs to only 1 loop/mesh with current I_j, the voltage drop across R_i is $I_j R_i$.

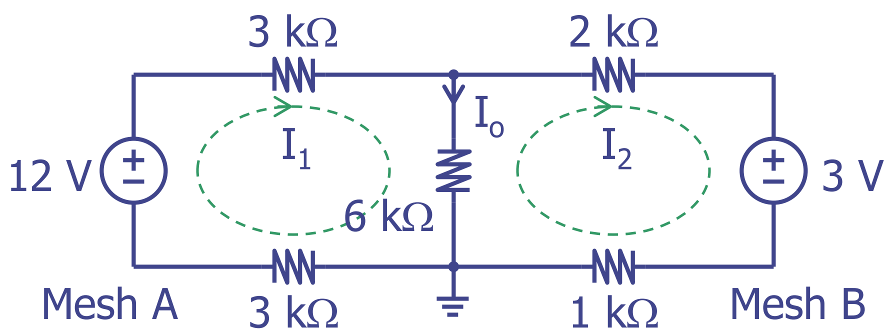
(3) For R_i that belongs to two loops/meshes with currents I_j and I_k, special care is needed to determine the voltage drop as $(\pm I_j \pm I_k)R_i$ according to the directions of I_j and I_k.

(4) Solve for the b–n+1 equations.

Note that using loop/mesh analysis always have **equal or more** equations to solve than using nodal analysis.

Example 2-27 (Optional)

Example 2-27: Solve for I_o.



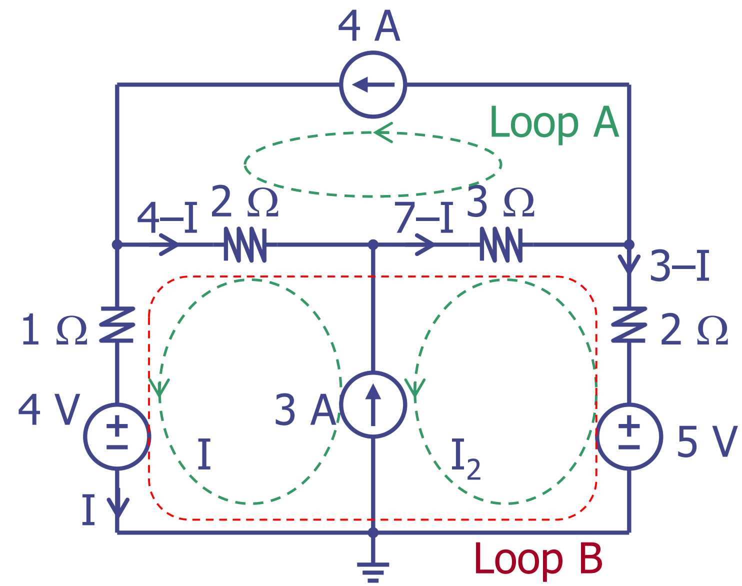
Soln.:

KVL of Mesh A:	$12 - 3k\times I_1 - 6k\times(I_1 - I_2) - 3k\times I_1 = 0$	(1)
KVL of Mesh B:	$3 + 2k\times I_2 + 6k\times(I_2 - I_1) + 1k\times I_2 = 0$	(2)
(1) $\Rightarrow$	$12k\times I_1 - 6k\times I_2 = 12$	(3)
(2) $\Rightarrow$	$6k\times I_1 - 9k\times I_2 = 3$	(4)
(3) – 2×(4) $\Rightarrow$	$12k\times I_2 = 6 \Rightarrow I_2 = 0.5$ mA	
I_2 in (3)/6 $\Rightarrow$	$2k\times I_1 = 2 + 1k\times 0.5 \Rightarrow I_1 = 1.25$ mA	
Finally,	$I_o = I_1 - I_2 = 0.75$ mA	

Ans. $I_1 = 1.25$ mA, $I_2 = 0.5$ mA, $I_o = 0.75$ mA

Example 2-28 (Optional)

Example 2-28: Solve for I.



Soln.:

Loop A is the mesh with the 4 A source, but KVL cannot be easily applied, nor to the meshes with the 3 A source. Hence, define Loop B (a superloop or supermesh) as shown. To facilitate computation, use KCL at the nodes to find the unknown currents first.

At the branch with 3 A source: $I - I_2 = 3 \Rightarrow I_2 = I - 3$

Loop B: $4 + 1\times I + 2\times(I-4) + 3\times(I-3-4) + 2\times(I-3) - 5 = 0$

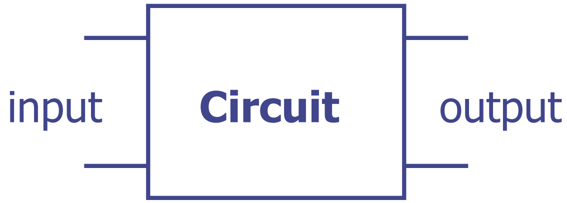
$\Rightarrow$ $4 + I + 2\times I - 8 + 3\times I - 21 + 2\times I - 6 - 5 = 0$

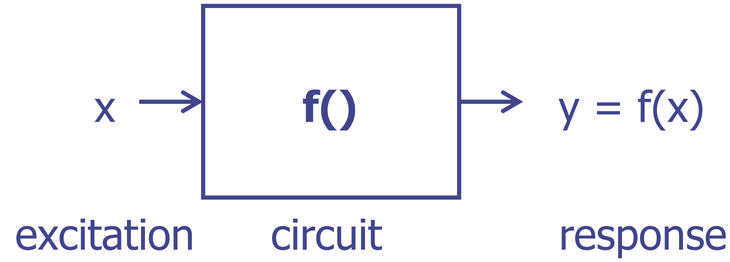
$\Rightarrow$ $8I = 36$ $\Rightarrow$ Ans. $I = 4.5$ A

The loop/mesh analysis is not straightforward and is not preferred.

2.4.3 Linearity

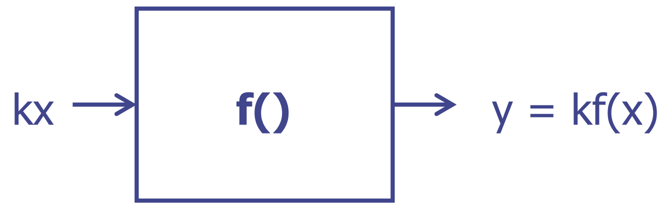
A circuit can be considered mathematically as a function, with an input and an output. The input is also known as the excitation, and the output is known as the response.





A circuit satisfies the property of homogeneity iff (if and only if)

$$f(kx) = kf(x)$$



The only function that can meet the homogeneity requirement is $f(x) = mx$, i.e., a straight line passing through the Origin.

Superposition and Linearity

A circuit satisfies the property of superposition iff (if and only if)

$$f(x_1+x_2) = f(x_1) + f(x_2)$$

$$x_1 + x_2 \longrightarrow \boxed{\mathbf{f()}} \longrightarrow y = f(x_1) + f(x_2)$$

A circuit is linear iff it satisfies both the properties of homogeneity and superposition, that is

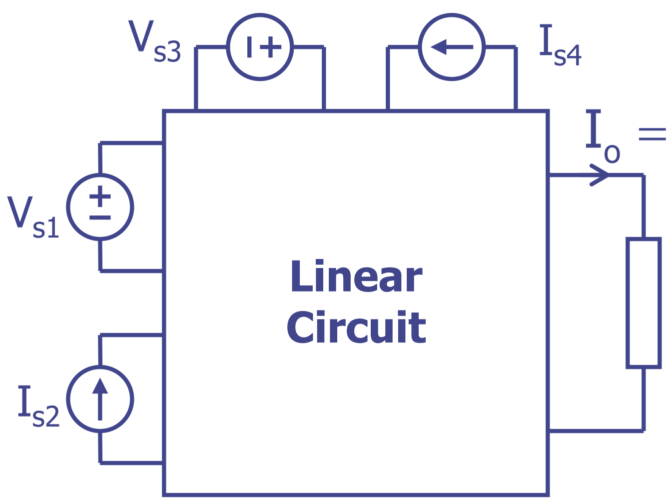
$$f(ax_1+bx_2) = af(x_1) + bf(x_2)$$

$$ax_1 + bx_2 \longrightarrow \boxed{\mathbf{f()}} \longrightarrow y = af(x_1) + bf(x_2)$$

Application of Superposition

In this course, we often deal with linear circuits (with important exceptions, e.g., diodes), and as such, superposition applies. Therefore, if a linear circuit contains multiple independent sources, the output voltage and/or the output current can be calculated by summing the contributions of each source acting alone.

When computing the individual contribution of a source, all **other independent sources** are set to zero: a voltage source becomes a short circuit (V_{si} = 0 V) and a current source becomes an open circuit (I_{sj} = 0 A). Yet, all **dependent sources** should remain operative.



$$I_o = I_o\big|_{V_{s1}} + I_o\big|_{I_{s2}} + I_o\big|_{V_{s3}} + I_o\big|_{I_{s4}}$$

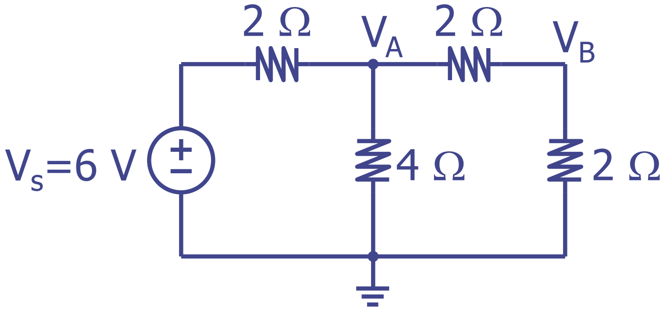
+

$$V_o = V_o\big|_{V_{s1}} + V_o\big|_{I_{s2}} + V_o\big|_{V_{s3}} + V_o\big|_{I_{s4}}$$

−

Example 2-29

Example 2-29: Compute V_A and V_B for V_s = 6 V and V_s = 12 V. Comment on the result.



Soln.:

$$V_A = \frac{4\,||\,4}{4\,||\,4+2} V_s = 0.5 V_s$$

$$V_B = \frac{2}{2+2} V_A = 0.5 V_A$$

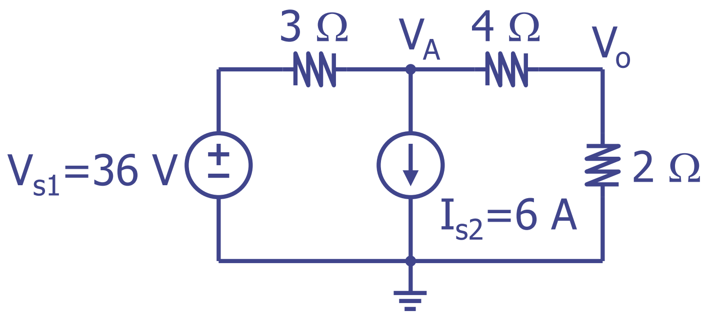
Therefore,

(1) for V_s = 6 V, V_A = 3 V, V_B = 1.5 V; and
(2) for V_s = 12 V, V_A = 6 V, V_B = 3 V.

Note that the node voltages are proportional to V_s, observing homogeneity.

Example 2-30

Example 2-30: Compute V_o by using nodal analysis and by using superposition.



Soln.:

(1) Nodal analysis:

KCL at V_A : $\dfrac{36 - V_A}{3} = 6 + \dfrac{V_A}{6}$

$\Rightarrow \quad 72 - 2V_A = 36 + V_A$

$\Rightarrow \quad V_A = 12V$

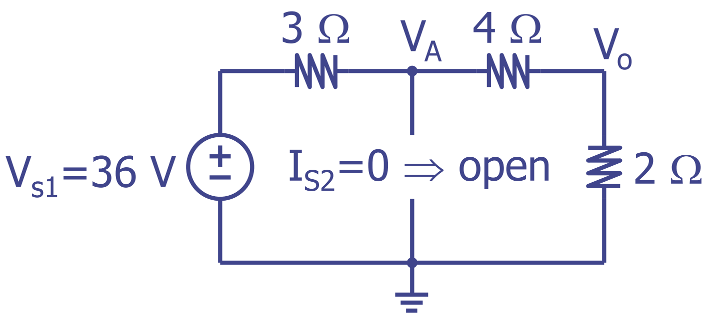
and $\quad V_o = 4$ V

Example 2-30 (cont.)

(2) Superposition:

(i) I_{s2} set to 0:

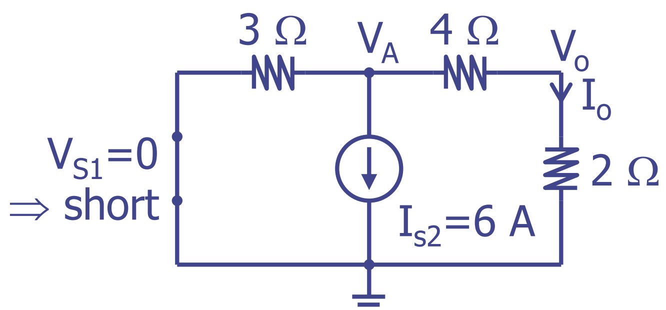
$$V_o\Big|_{I_{s2}=0} = \frac{2}{3+4+2}\times 36 = 8V$$



(ii) V_{s1} set to 0:

$$I_o\Big|_{V_{s1}=0} = \frac{3}{3+6}\times(-6) = -2A$$

$$V_o\Big|_{V_{s1}=0} = -2A\times 2\Omega = -4V$$

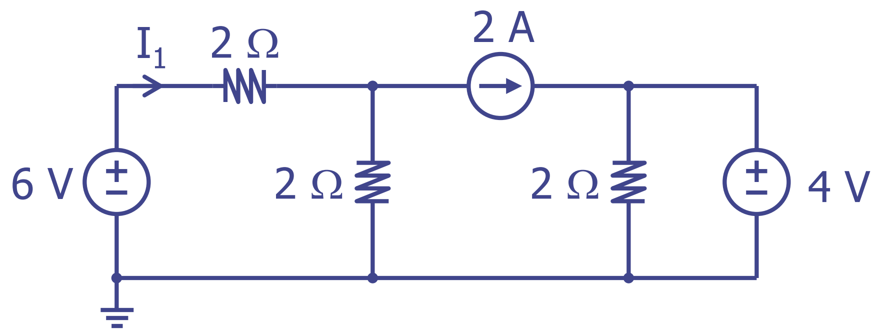


Finally, add the two contributions together to get

$$V_o = V_o\Big|_{V_{s1}=0} + V_o\Big|_{I_{s2}=0} = -4V + 8V = 4V$$

Example 2-31

Example 2-31: Compute I_1 (superposition with tricky circuitry).

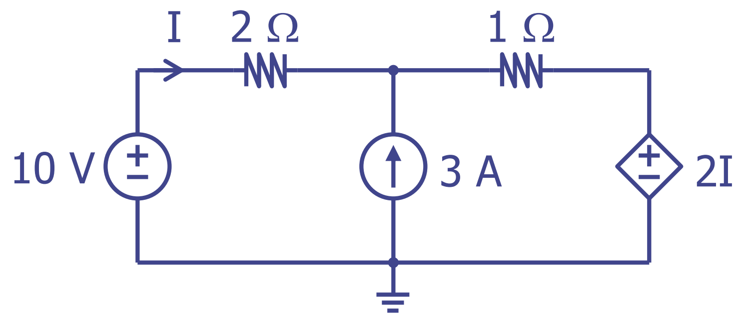


Soln.: Note that the parallel combination of 2 Ω||4 V is in series with the 2 A source, and plays no part in determining I_1.

$$
\begin{aligned}
I_1 &= I_1\big|_{6V} + I_1\big|_{2A} \\
&= \frac{6V}{2\Omega + 2\Omega} + \frac{2\Omega}{2\Omega + 2\Omega} \times 2A \\
&= 1.5A + 1A \\
&= 2.5A
\end{aligned}
$$

Example 2-32 (1)

Example 2-32: Use superposition to find I (note the dependent source).



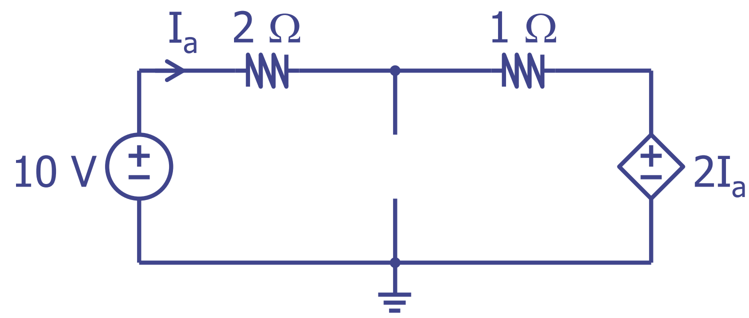
Soln.:

(1) Consider I due to the 10 V source first (= I_a), and remember that the dependent source 2I (now $2I_a$) remains operative.

Apply KVL to the loop:

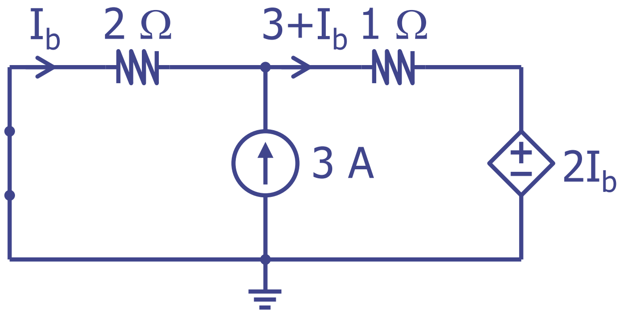
$$10 = 2I_a + I_a + 2I_a$$

$$\Rightarrow \quad I_a = 2\ \text{A}$$



Example 2-32 (2)

(2) Next, consider I due to the 3 A source (= I_b), and remember that the dependent source 2I (now $2I_b$) remains operative.



Apply KVL to the loop:

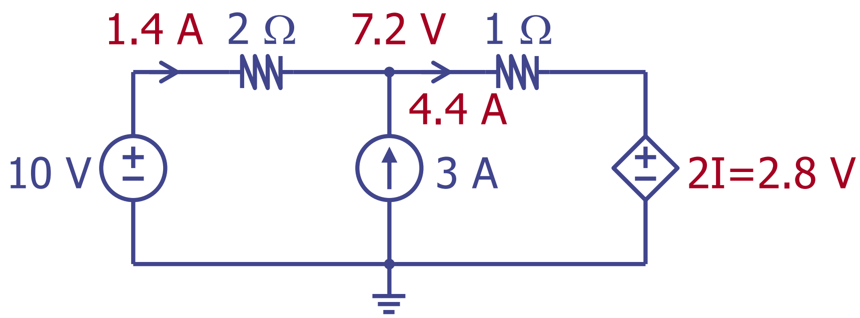
$$2I_b + (3 + I_b) + 2I_b = 0$$

$$\Rightarrow \quad I_b = -0.6 \text{ A}$$

Hence, the answer is

$$I = I_a + I_b = 2 - 0.6 = 1.4 \text{ A}$$

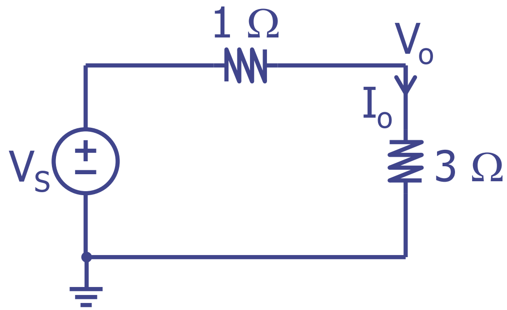
Checking:



Superposition not applicable to Power

Power is a square function of voltage and current ($P \propto V^2, I^2$), and as it is not a linear function of V and I, it does not obey superposition.

Example 2-33: Find the power absorbed by the 3 Ω resistor for V_s = 8 V and V_s = 16 V.



Soln.:

(1) For V_s = 8 V, I_o = 2 A, and $P_{3\Omega} = 2^2 \times 3 = 12$ W

(2) For V_s = 16 V, I_o = 4 A, and $P_{3\Omega} = 4^2 \times 3 = 48$ W

Note that V_s increases by 2 times, but the power is increased by 4 times.

Chapter 2: Resistive Networks and DC Analysis

2.1 Circuit Terminology

2.2 Circuit Laws

  2.2.1 Kirchhoff's Current Law

  2.2.2 Kirchhoff's Voltage Law

2.3 Resistive Network

  2.3.1 Resistors in Series and in Parallel

  2.3.2 Voltage and Current Dividers

2.4 Circuit Analysis

  2.4.1 Nodal Analysis

  2.4.2 Loop and Mesh Analysis

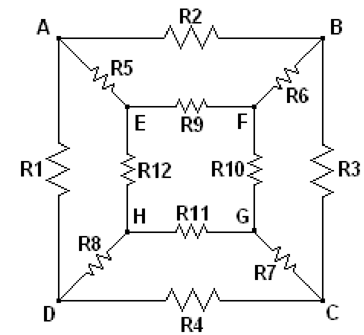
  2.4.3 Superposition

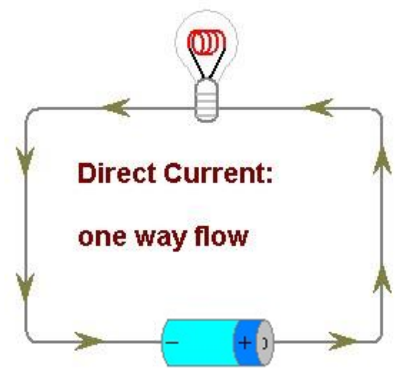
2.5 Maximum Power Transfer & High-Voltage Transmission

2.6 Equivalence and Source Transformation

  2.6.1 Thevenin's and Norton's Theorems

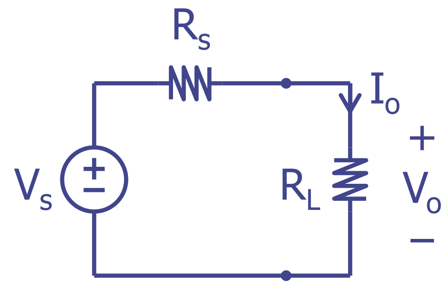
  2.6.2 General Proof





2.5 Maximum Power Transfer (1)

Let us consider a voltage source V_s with source resistance R_s (= output resistance of V_s) driving a load R_L:



The power dissipated in (or delivered to) R_L is

$$P_L = V_o I_o = \frac{R_L}{R_s + R_L} V_s \times \frac{V_s}{R_s + R_L}$$

$$= \frac{R_L}{(R_s + R_L)^2} V_s^{\,2}$$

Maximum Power Transfer (2)

To obtain the maximum power dissipated in R_L, differentiate P_L w.r.t. (with respect to) R_L and set the result to zero:

$$\frac{dP_L}{dR_L} = 0 \qquad \Rightarrow V_s^{\,2} \frac{(R_s + R_L)^2 - R_L \times 2 \times (R_s + R_L)}{(R_s + R_L)^4} = 0$$

gives

$$R_s^{\,2} + 2R_sR_L + R_L^{\,2} - 2R_sR_L - 2R_L^{\,2} = 0$$

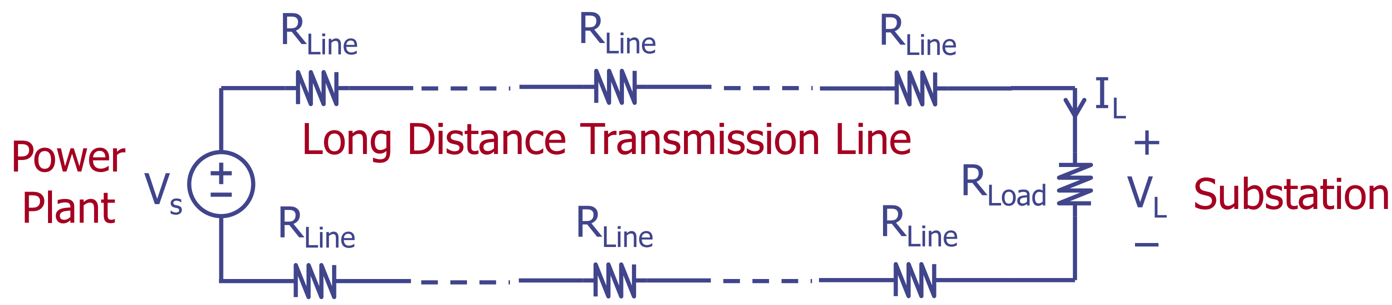
$$\Rightarrow \qquad R_L = R_s$$

The maximum power $P_{L(max)}$ that can be obtained from a source V_s is when the load resistance R_L is matched to the source resistance R_s ($R_L = R_s$), and

$$P_{L(max)} \quad = \frac{R_s}{(R_s + R_s)^2} V_s^{\,2} = \frac{1}{4} \frac{V_s^{\,2}}{R_s}$$

High-Voltage Transmission

Let us consider a different objective: to deliver a given amount of power from the power plant to the substation while minimizing the power loss in the long distance transmission line.



The power delivered to R_{Load} is

$$P_{Load} = V_L I_L = \text{Given}$$

The power loss in the transmission line is

$$P_{Loss} = I_L^2 \sum R_{Line} = \left(\frac{P_{Load}}{V_L}\right)^2 \sum R_{Line} \propto \frac{1}{V_L^2}$$

High voltage $\Rightarrow$ lower transmission loss. The highest transmission voltage now exceeds 1 MV, with P_{Load} rated over 10 GW in China.

https://en.wikipedia.org/wiki/Ultra-high-voltage_electricity_transmission_in_China

Chapter 2: Resistive Networks and DC Analysis

2.1 Circuit Terminology

2.2 Circuit Laws

- 2.2.1 Kirchhoff's Current Law
- 2.2.2 Kirchhoff's Voltage Law

2.3 Resistive Network

- 2.3.1 Resistors in Series and in Parallel
- 2.3.2 Voltage and Current Dividers

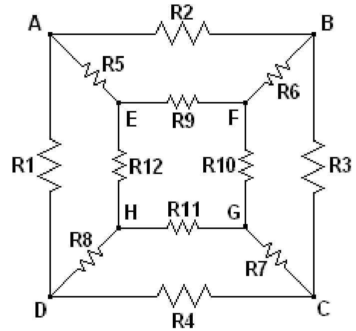
2.4 Circuit Analysis

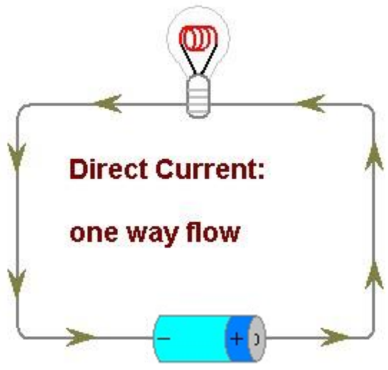
- 2.4.1 Nodal Analysis
- 2.4.2 Loop and Mesh Analysis
- 2.4.3 Superposition

2.5 Maximum Power Transfer & High-Voltage Transmission

2.6 Equivalence and Source Transformation

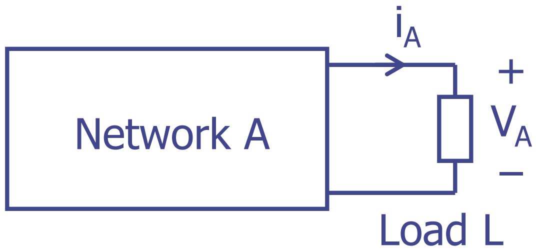
- **2.6.1 Thevenin's and Norton's Theorems**
- **2.6.2 General Proof**

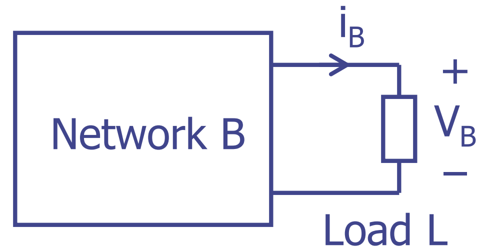




2.6 Equivalence

Two resistive 1-port networks are equivalent if and only if they have the same current-voltage (I-V) characteristics across their respective terminal-pairs for ALL loads (including sources).

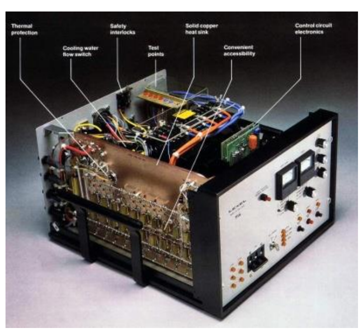




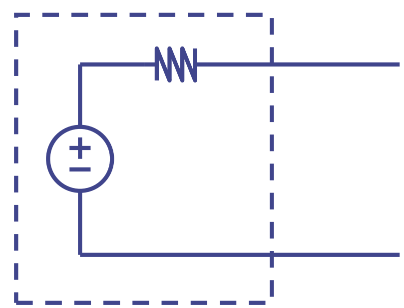
If $I_A = I_B$ and $V_A = V_B$ for all load L, then network A and Network B are equivalent.

Hence, a complex network (Network A) can be replaced by a simple equivalent network (Network B), and the analysis can be simplified.

Power supply

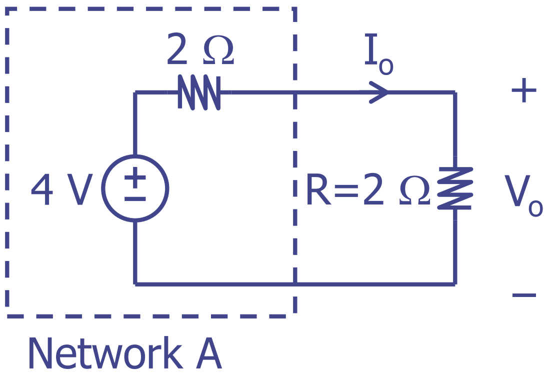


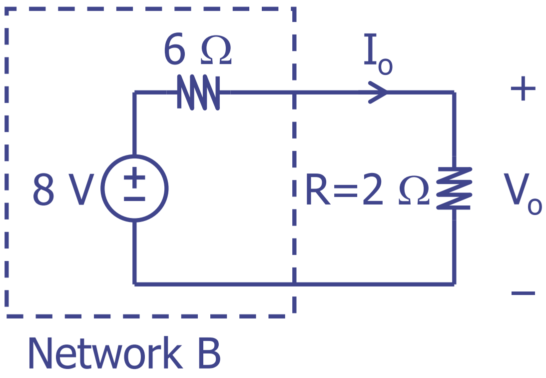
$\equiv$



Example 2-34

Example 2-34: Is Network A equivalent to Network B?





Soln.:

For R = 2 Ω, the load voltage and load current for both Network A and Network B are V_o = 2 V and I_o = 1 A. However, for R = 6 Ω, Network A gives: V_o = 3 V
and I_o = 0.5 A

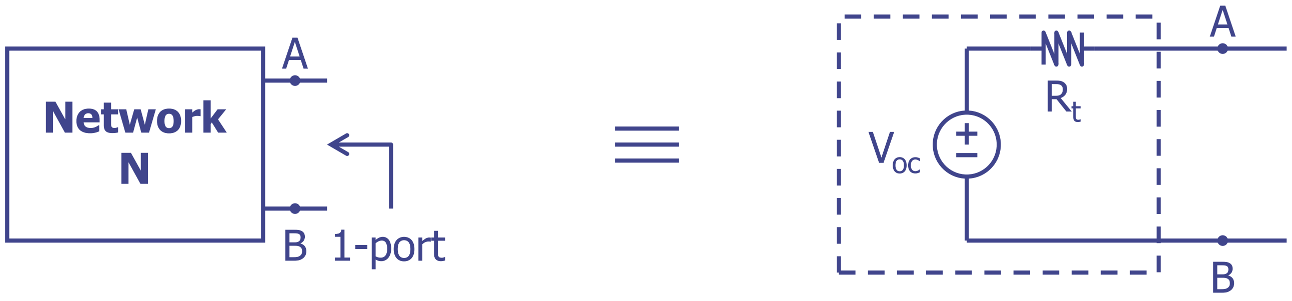
but Network B gives: V_o = 4 V
and I_o = 0.75 A

Therefore, Network A is not equivalent to Network B.

2.6.1 Thevenin's Theorem

Thevenin's Theorem:

A linear circuit with a terminal-pair (N) can be replaced by a series combination of an ideal voltage source V_{oc} and a resistor R_t, where V_{oc} is the open-circuit voltage of N and R_t is the equivalent resistance looking into N with all independent sources set to zero. All dependent sources should remain operative.

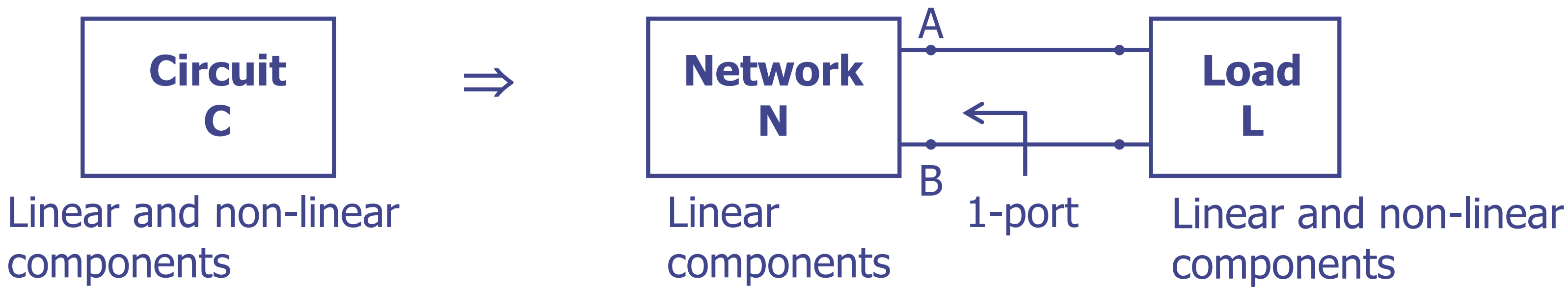


Thevenin's equivalent circuit of N

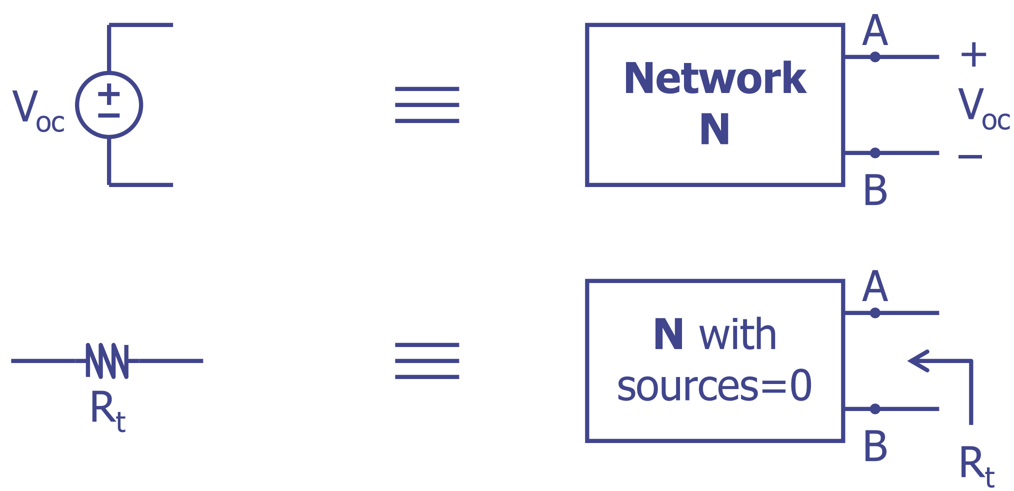
1. V_{oc} is also known as Thevenin's equivalent source.
2. R_t is Thevenin's resistance, or the output resistance of N.

Computing V_{oc} and R_t

Figuratively, we have a complicated circuit C that can be divided into two parts, a linear network N and a load L.



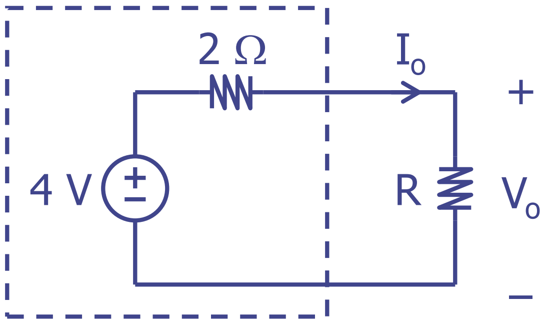
Thevenin's theorem states that:



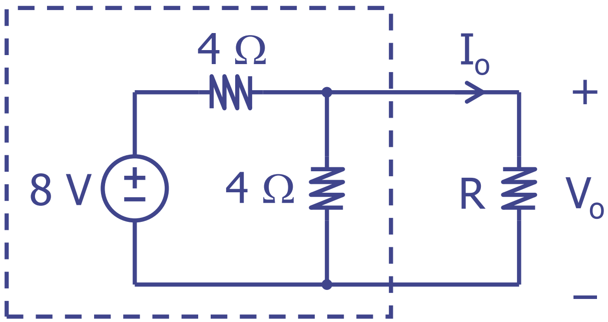
(Dependent sources remain operative)

Example 2-35

Example 2-35: Is Network A equivalent to Network B?

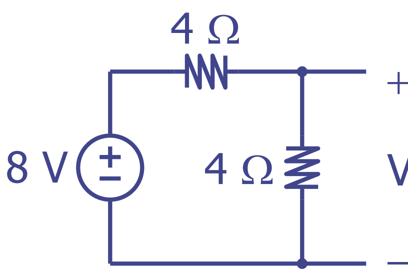


Network A

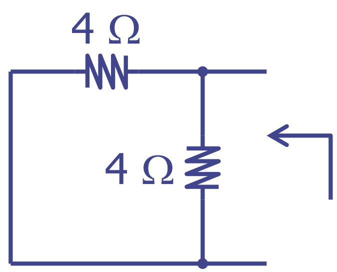


Network B

Soln.: Construct Thevenin's equivalent circuit of Network B:



$$V_{oc} = \frac{4}{4+4} \times 8 = 4\text{V}$$



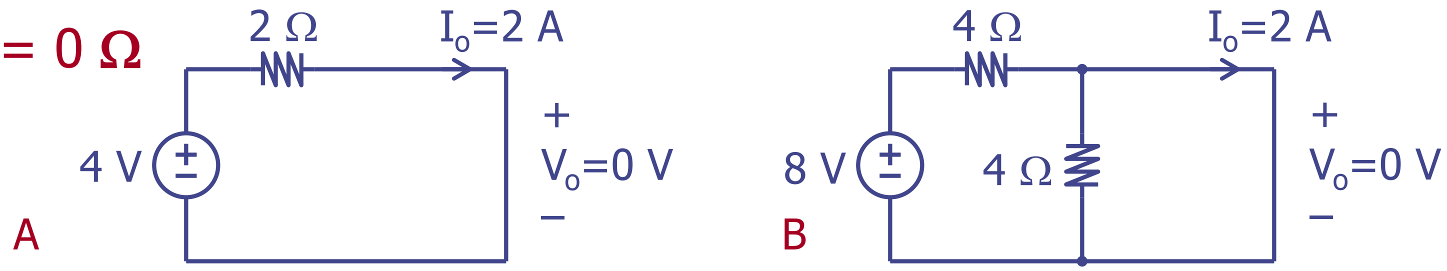
$$R_t = 4 \,||\, 4 = 2\Omega$$

Now, Network B is modeled as V_{oc} = 4 V in series with R_t = 2 **Ω**, and is the same as Network A; hence, they are equivalent.

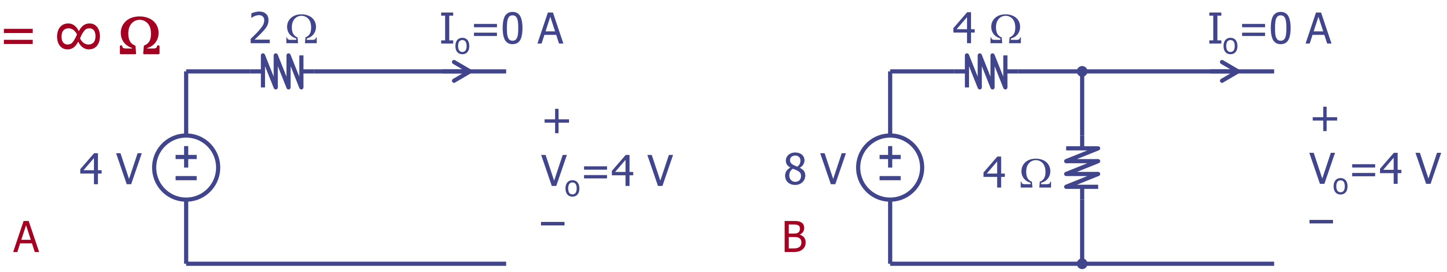
Example 2-35 (cont.)

An alternative way to demonstrate equivalence is to consider the load with R = 0 Ω and R = ∞ Ω.

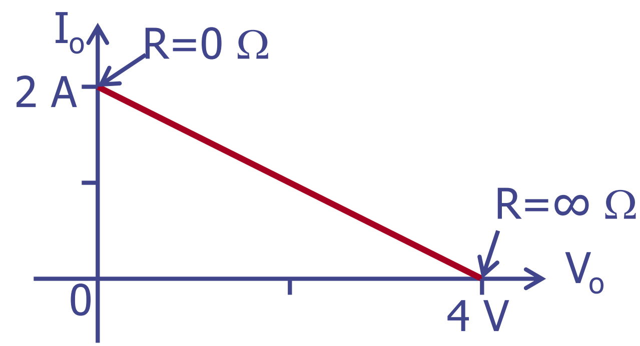
R = 0 Ω



R = ∞ Ω

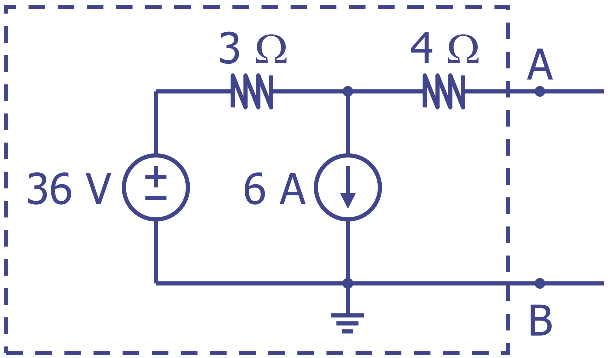


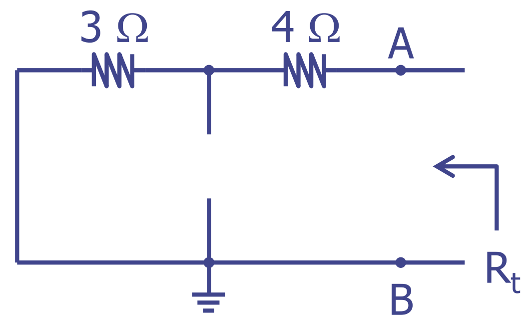
Both give the same I_o-V_o plot for all R:



Example 2-36

Example 2-36: Find V_{oc} and R_t of the following circuit.



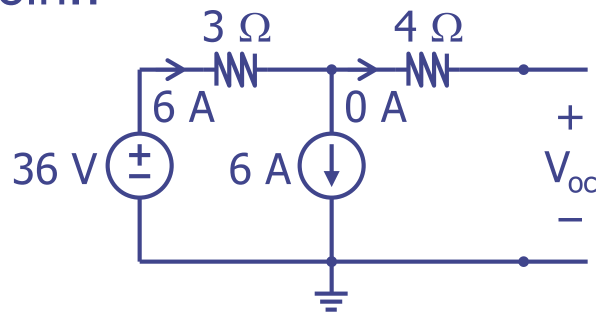


$36\text{ V} \rightarrow 0 \Rightarrow$ shorted

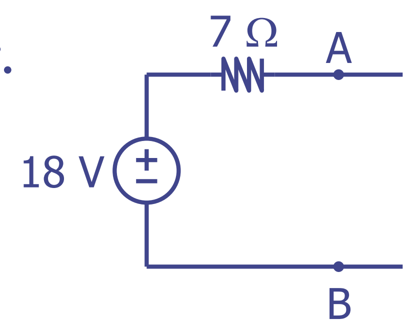
$6\text{ A} \rightarrow 0 \Rightarrow$ open

gives $R_t = 7\ \Omega$

Soln.:



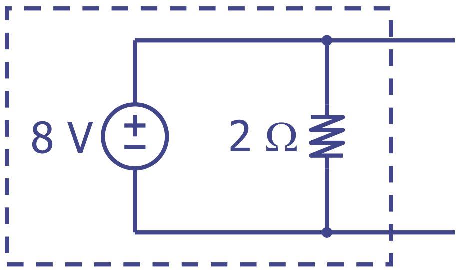
$\therefore$



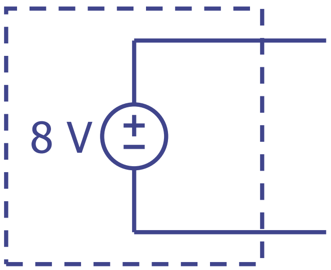
$$V_{oc} = 36 - 6\times 3 + 0\times 4 = 18\text{ V}$$

Example 2-37

Example 2-37: Is Network A equivalent to Network B?

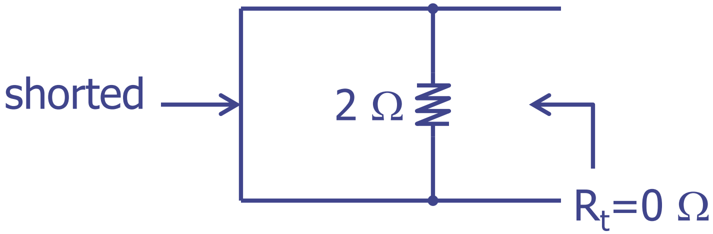


Network A



Network B

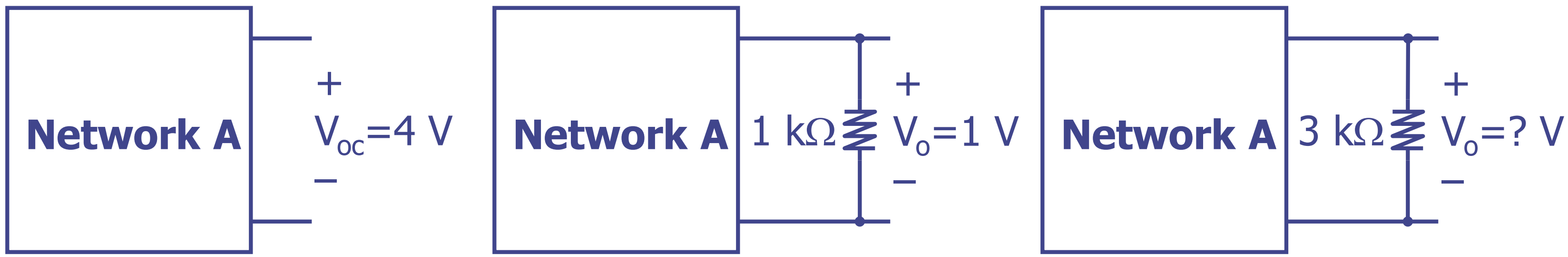
Soln.: For Thevenin's equivalent circuit of Network A, clearly, V_{oc} = 8 V and R_t = 0 **Ω**. Hence, Network A is equivalent to Network B.



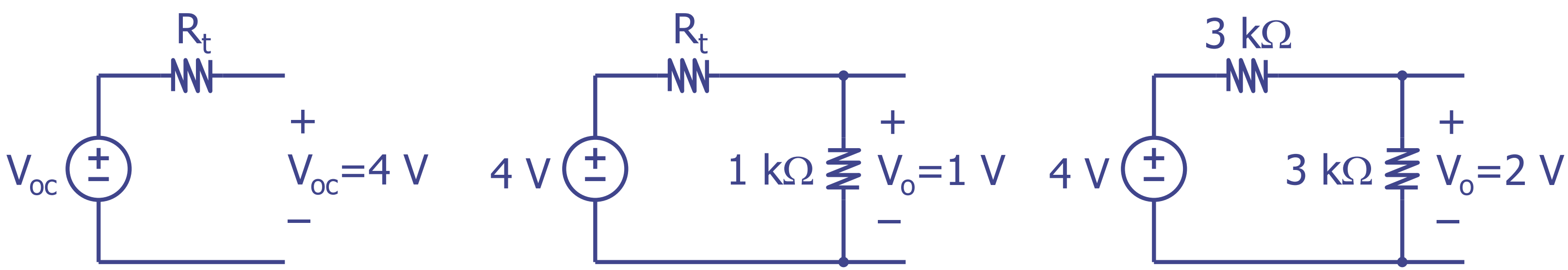
Note that any resistors in parallel with an ideal voltage source can be neglected from calculating other circuit variables.

Example 2-38

Example 2-38: Find the output voltage V_o when R_L changes from 1 to 3 kΩ.



Soln.:



Given

V_{oc} = 4 V

Clearly,

R_t = 3 kΩ

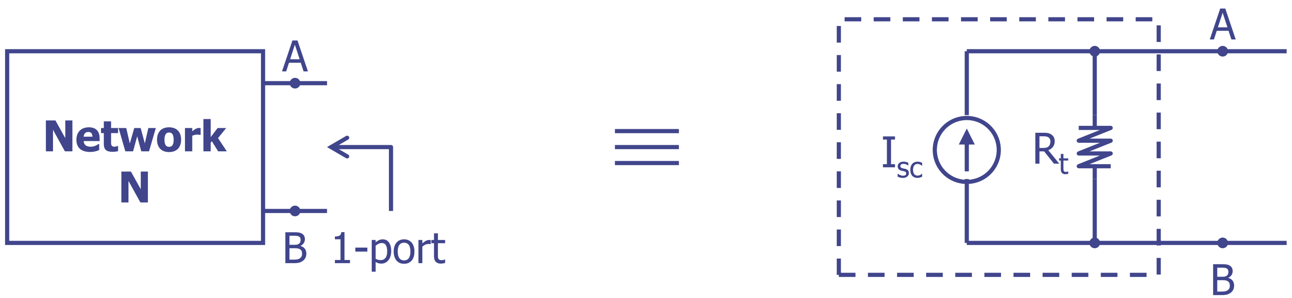
Ans:

V_o = 2 V

Norton's Theorem

Norton's Theorem:

A linear circuit with a terminal-pair (N) can be replaced by a parallel combination of an ideal current source I_{sc} and a resistor R_t, where I_{sc} is the short-circuit current of N and R_t is the equivalent resistance looking into N with all independent sources set to zero. All dependent sources should remain operative.



Norton's equivalent circuit of N

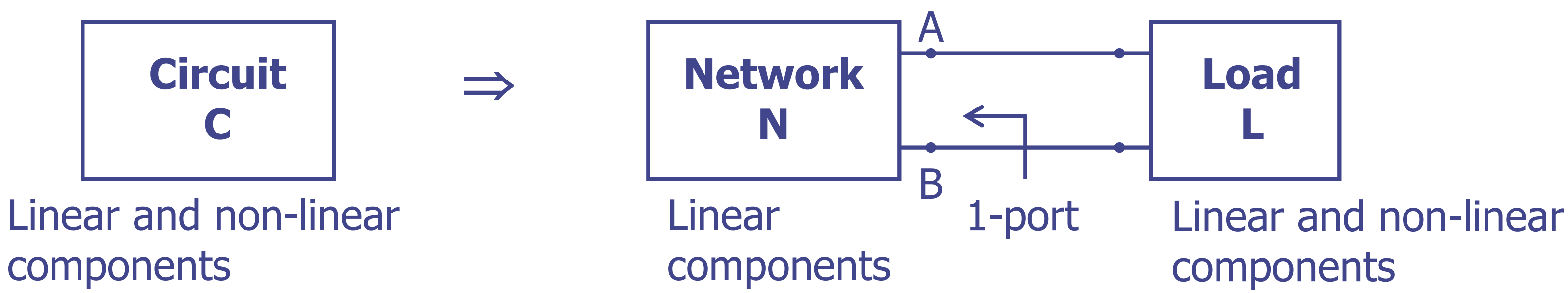
Points to note:

(1) The equivalent resistance R_t is the same for both Thevenin's and Norton's equivalent circuits.

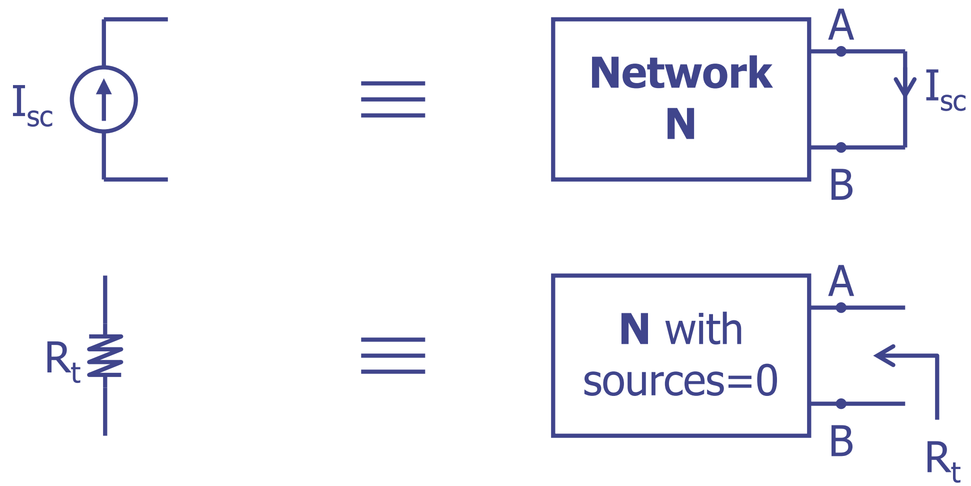
(2) It is easy to show that $V_{oc} = I_{sc}R_t$.

Computing I_{sc} and R_t

Figuratively, we have a complicated circuit C that can be divided into two parts, a linear network N and a load L.



Norton's theorem states that:

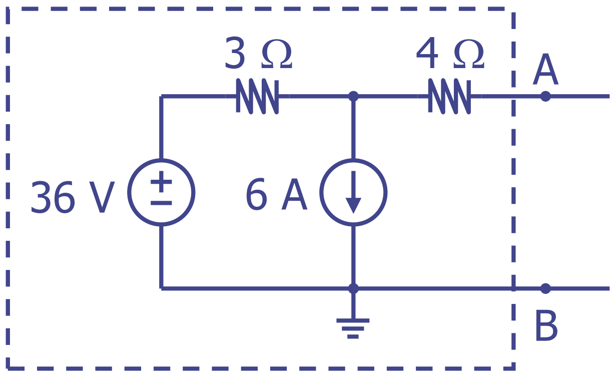


(Note the direction of I_{sc})

(Dependent sources remain operative)

Example 2-39

Example 2-39: Find I_{sc} and R_t of the circuit in Example 2-36.



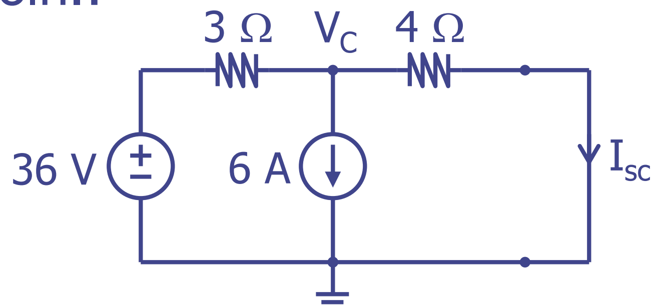
KCL at V_C: $$\frac{36 - V_C}{3} = 6 + \frac{V_C}{4}$$

$$\Rightarrow \quad 144 - 4V_C = 72 + 3V_C$$

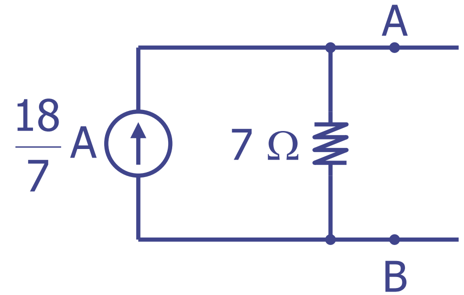
$$\Rightarrow \quad V_C = \frac{72}{7}\,\text{V}$$

$$\Rightarrow \quad I_{sc} = \frac{V_C}{4} = \frac{18}{7}\,\text{A}$$

Soln.:

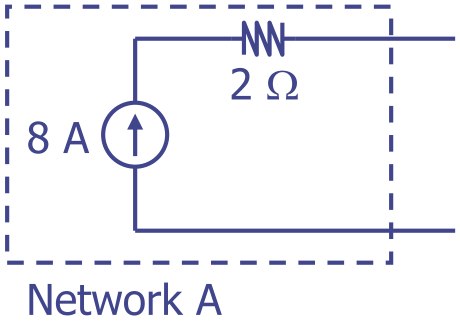


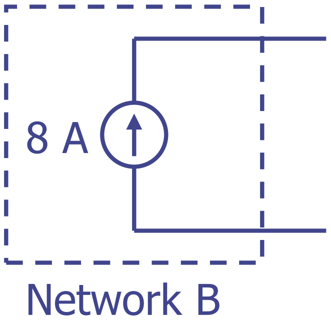
Recall that R_t = 7 Ω, Norton's equivalent is therefore



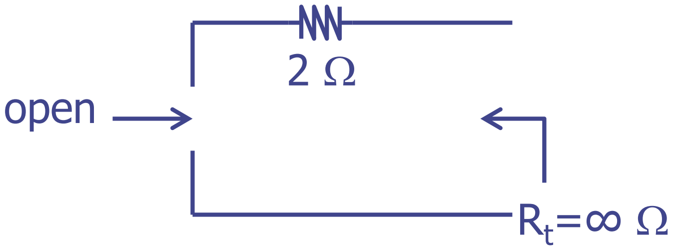
Example 2-40

Example 2-40: Is Network A equivalent to Network B?





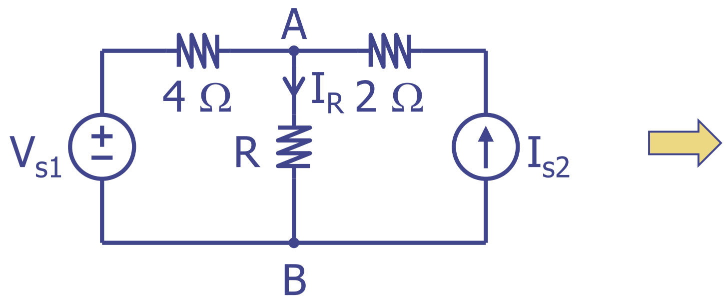
Soln.: For Norton's equivalent circuit of Network A, clearly, $I_{sc} = 8$ A and $R_t = \infty\ \Omega$. Hence, Network A is equivalent to Network B.

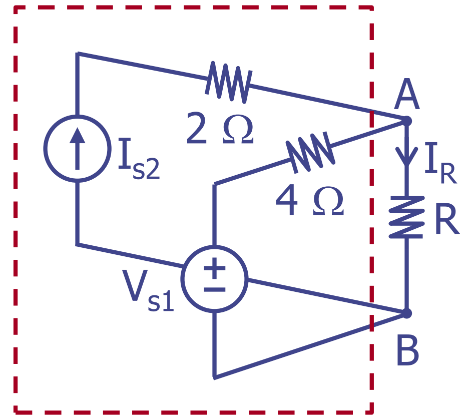


Note that any resistors in series with an ideal current source can be neglected from calculating other circuit variables.

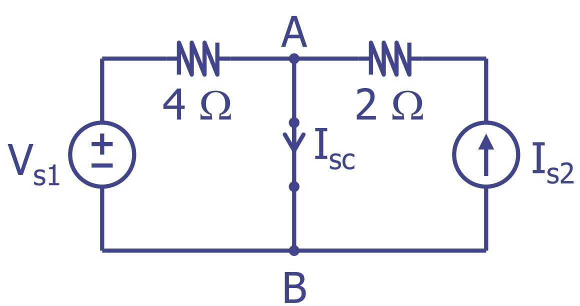
Example 2-41

Example 2-41: Both V_{s1} and I_{s2} are unknown. If R = 0 Ω, then I_R = 4 A. Find I_R when R = 4 Ω.

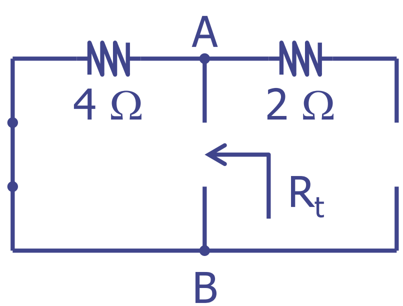




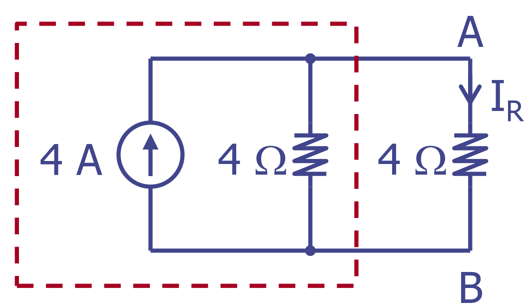
Soln.: Let R be the load, and find Norton's equivalent circuit of the remaining circuit first.



For R = 0 Ω, I_R = 4 A = I_{sc}.



R_t = 4 Ω

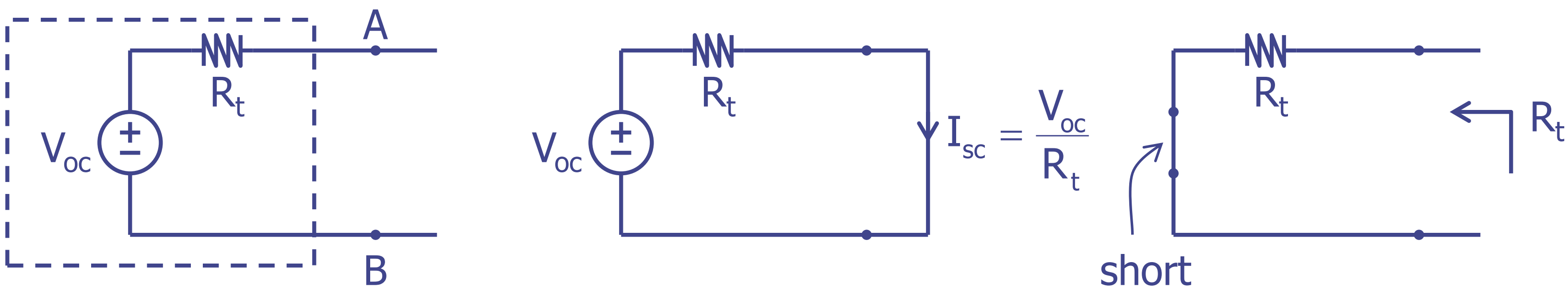


For R = 4 Ω, I_R = 2 A.

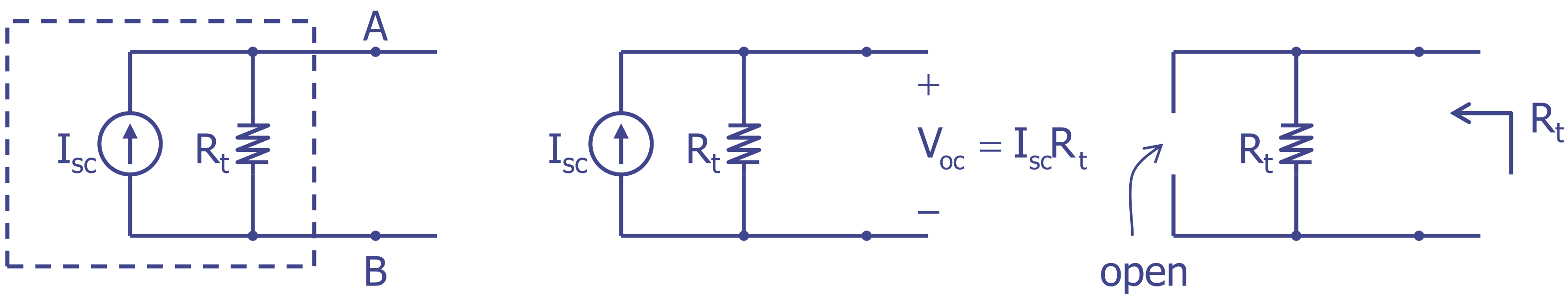
Transformation Between Thevenin's and Norton's Equivalent Circuits

A linear network can be described by either its Thevenin's or Norton's equivalent circuit. The validity of one will prove the other. They can also be transformed (source transformation) into one another.

From Thevenin's equivalent, find Norton's equivalent:



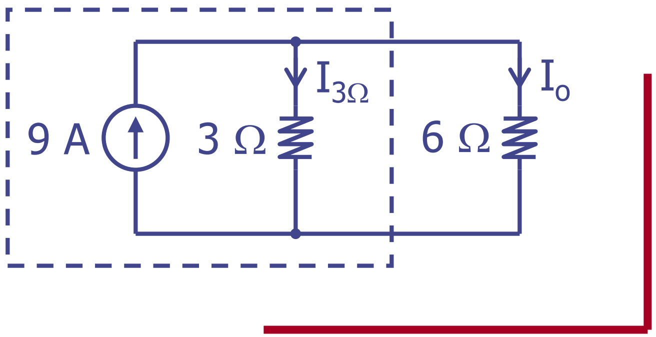
From Norton's equivalent, find Thevenin's equivalent:



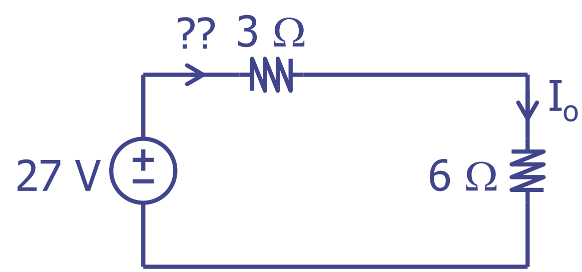
Equivalent for External Components Only

Source transformation can ONLY be employed to compute voltages and currents EXTERNAL TO Thevenin's or Norton's equivalent circuits.

Example 2-42: Demonstrate that source transformation can be used to compute I_o but not $I_{3\Omega}$.



Perform source transformation:



Soln.: From current division, we have

$I_{3\Omega} = 6$ A

$I_o = 3$ A

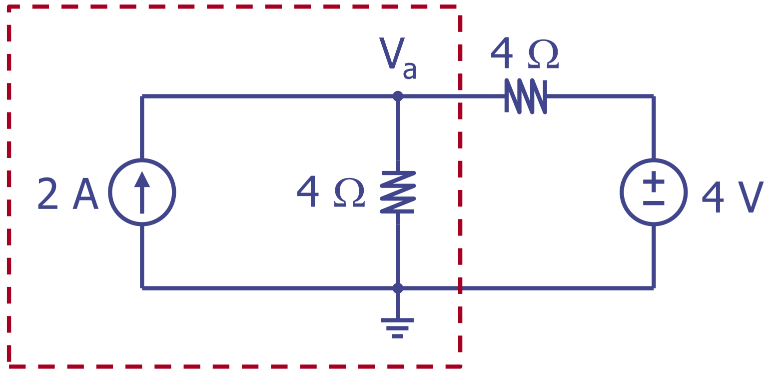
Clearly,

$I_o = 3$ A

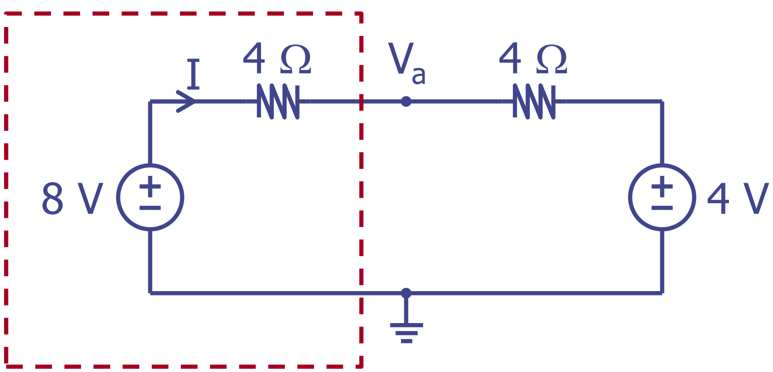
and $I_{3\Omega}$ is not the same as before.

Example 2-43 (1)

Example 2-43: Use source transformation to solve for V_a.



Soln.: One way is to obtain the Thevenin's equivalent circuit of 2 A||4 Ω:

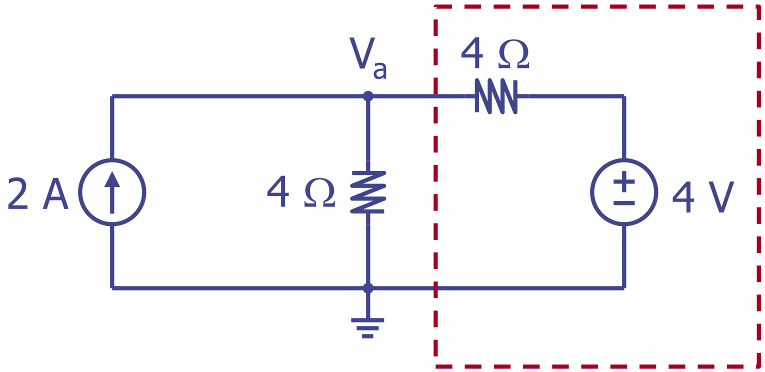


$$I = \frac{8-4}{4+4} = 0.5\text{A}$$

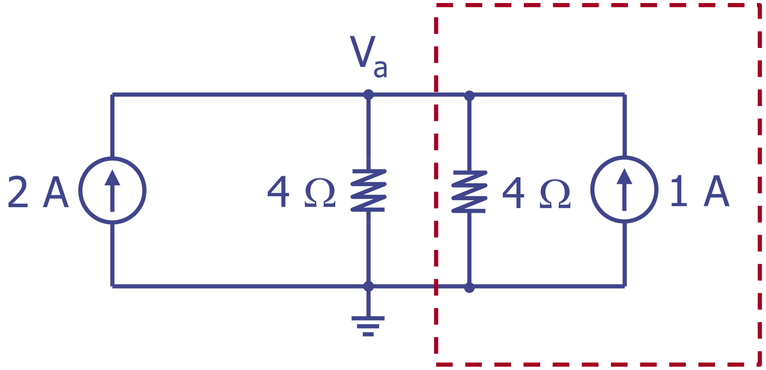
$$V_a = 8 - 4 \times 0.5 = 6\text{V}$$

Example 2-43 (2)

Example 2-43: Use source transformation to solve for V_a.



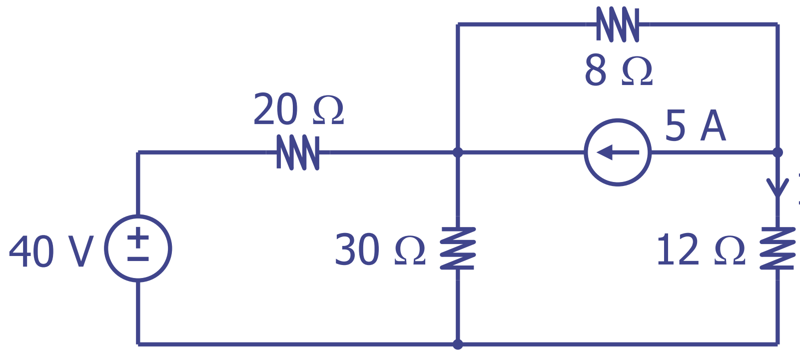
Soln.: A simpler way is to obtain the Norton's equivalent circuit of 4 V + 4 Ω on the right:



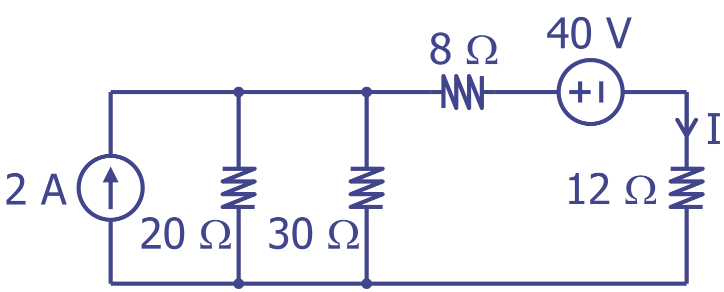
$$\begin{aligned} V_a &= (2\text{ A} + 1\text{ A})(4\ \Omega \| 4\ \Omega) \\ &= 3\text{ A} \times 2\ \Omega \\ &= 6\text{ V} \end{aligned}$$

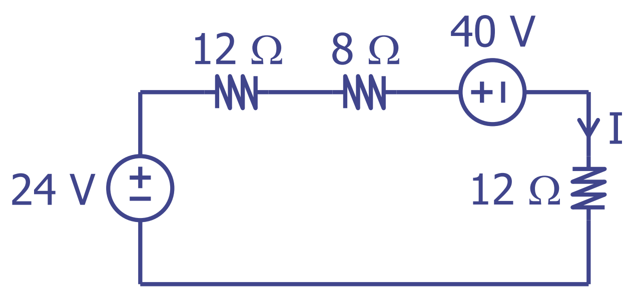
Example 2-44

Example 2-44: Use source transformation to solve for I.



Soln.: Obtain Norton's equivalent for 40 V + 20 **Ω** and Thevenin's equivalent for 5A||8 **Ω** first, and then Thevenin's equivalent for 2 A||20 **Ω**||30 **Ω**:





$$I = \frac{-16V}{32\Omega} = -0.5A$$

Example 2-45 (1)

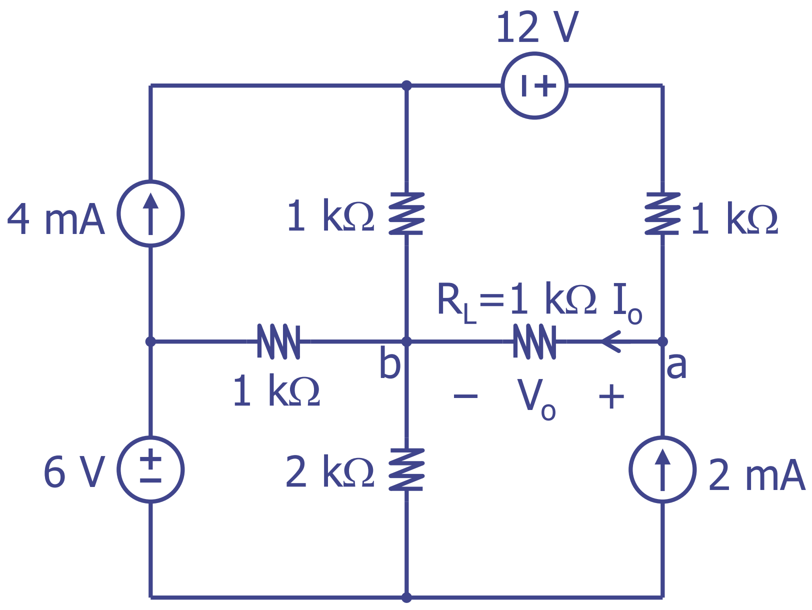
Example 2-45: Thevenin's equivalent circuit

(a) Find the open circuit voltage V_{oc} at the terminal pair a-b.

(b) Show that the Thevenin's resistance R_t at a-b is 2 kΩ.

(c) Find the short circuit current I_{sc} at the terminal-pair a-b.

(d) If the load is changed from 1 kΩ to 2 kΩ, find the new I_o.



Example 2-45 (2)

Soln.: (a) A formal solution involves writing KCL equations to solve. However, good observations may help to solve this problem faster. Here, make use of the currents of the current sources to find voltage drops, and work out the total voltage drop from V_a to V_b.

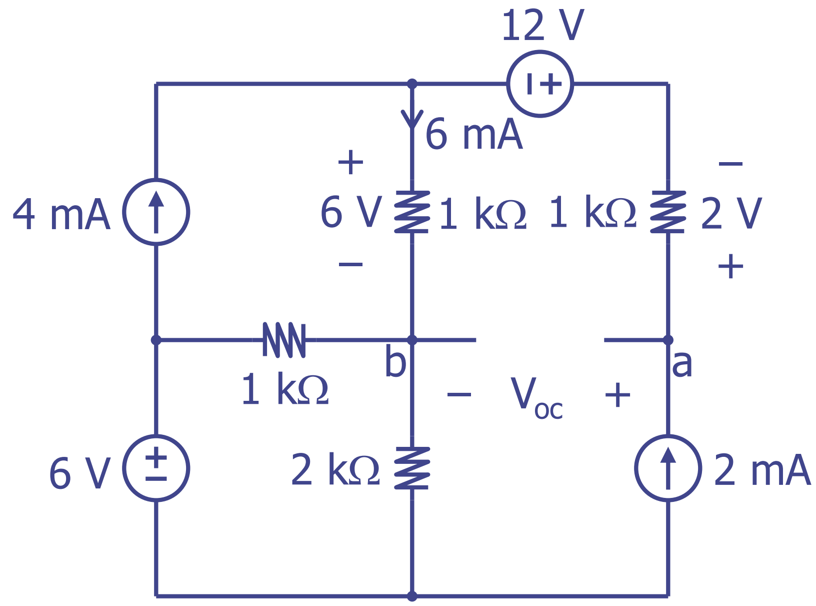
Note that

$$V_{oc} = V_a - V_b$$

and $V_a = V_b + 6\text{ V} + 12\text{ V} + 2\text{ V}$

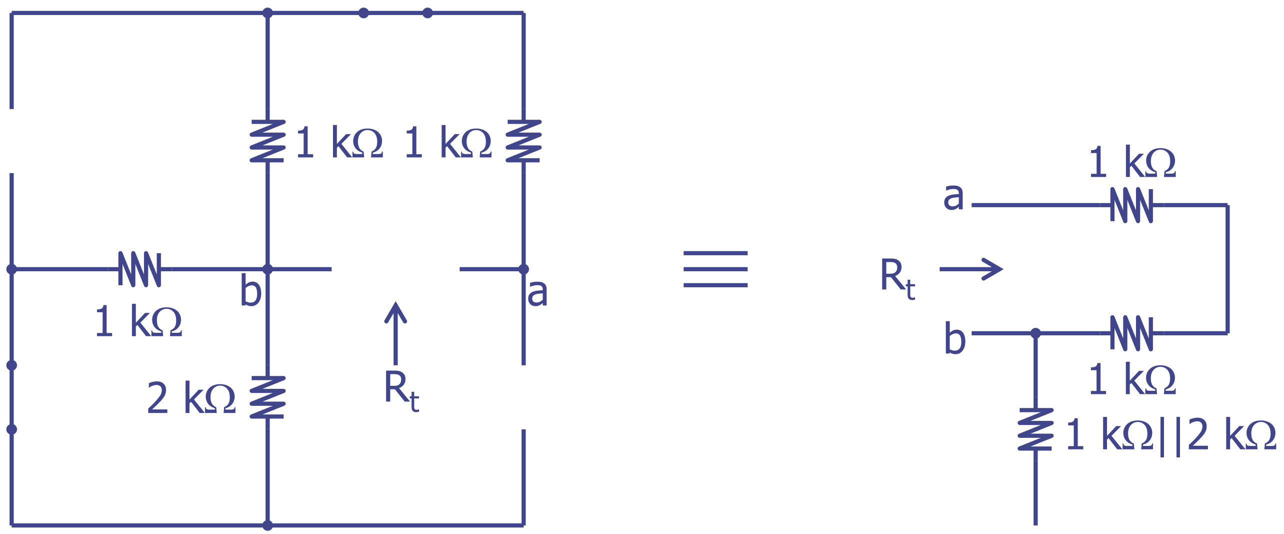
Hence,

$$V_{oc} = 20\text{ V}$$



Example 2-45 (3)

(b) For computing R_t, set all sources to zero (voltage sources shorted and current sources open).



Clearly, $R_t = 2\ \mathrm{k\Omega}$.

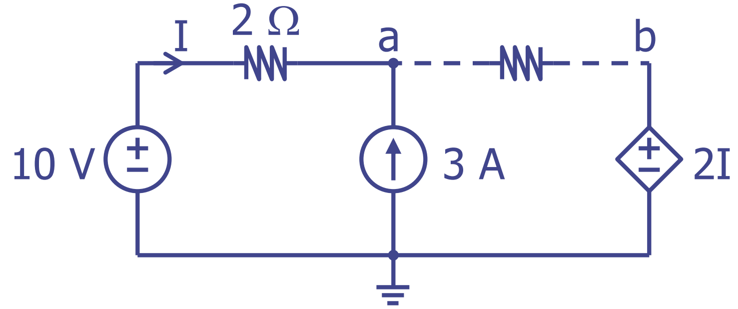
(c) $I_{sc} = V_{oc}/R_t = 20\ \mathrm{V}/2\ \mathrm{k\Omega} = 10\ \mathrm{mA}$.

(d) For $R_L = 2\ \mathrm{k\Omega}$,

$$I_o = V_{oc}/(R_t + R_L) = 20\ \mathrm{V}/(2\ \mathrm{k\Omega} + 2\ \mathrm{k\Omega}) = 5\ \mathrm{mA}.$$

Example 2-46 (1)

Example 2-46: Find Thevenin's equivalent circuit at a-b (equivalent circuit with dependent source).



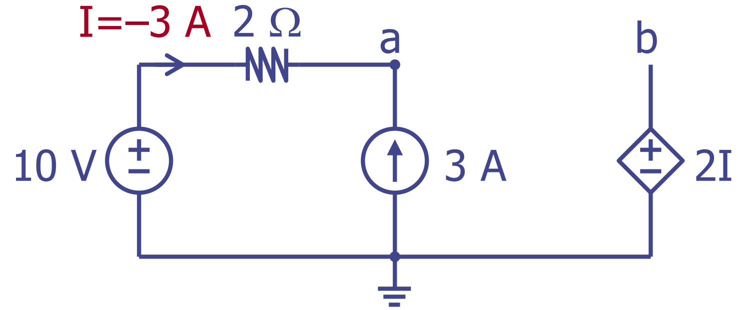
Soln.: (1) To find V_{oc} at a-b, note that

$$I = -3\text{ A}$$

$$V_a = 10 - (-3)\times 2 = 16\text{ V}$$

$$V_b = 2\times(-3) = -6\text{ V}$$

$$V_{oc} = V_a - V_b = 22\text{ V}$$



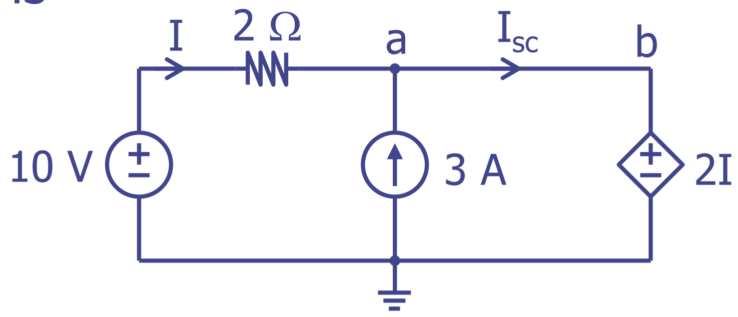
Example 2-46 (2)

(2) To find R_t is a little bit tricky (but can be done directly). We may find I_{sc} first, and $R_t = V_{oc}/I_{sc}$.

Now, the KVL equation relating V_a is

$$V_a = V_b = 10 - 2I = 2I$$

$\Rightarrow$ $I = 2.5$ A



Then, the KCL equation at V_a is

$$I + 3 = I_{sc}$$

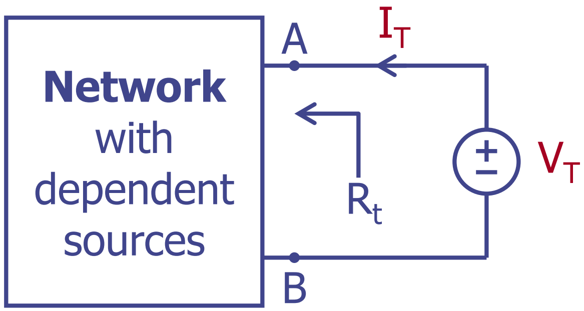
$\Rightarrow$ $I_{sc} = 5.5$ A

$\therefore$ $R_t = V_{oc}/I_{sc} = 22\text{ V}/5.5\text{ A} = 4\ \Omega$

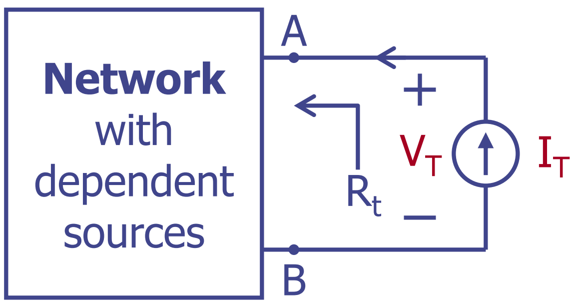
Example 2-46 (3)

(3) To find R_t of a network with one or more dependent sources.

(i) We may apply a test voltage V_T across A-B. Find the current I_T flows into the network at node A. Then, R_t is given by $R_t = V_T/I_T$.



(ii) Alternatively, we may apply a test current I_T flows into node A. Find the voltage V_T across terminals A-B. Again, R_t is given by $R_t = V_T/I_T$.



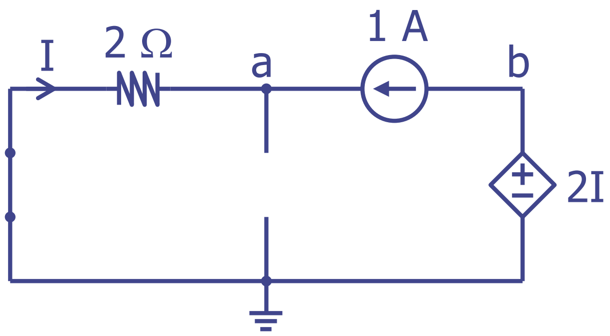
V_T and I_T can be set to 1 V and 1 A to simplify the calculation.

Example 2-46 (4)

Now, apply 1 A across node a-b.

Obviously, $I = -1$ A

Then, from the KVL equation at the outer loop



$$2I + V_{ab} + 2I = 0$$

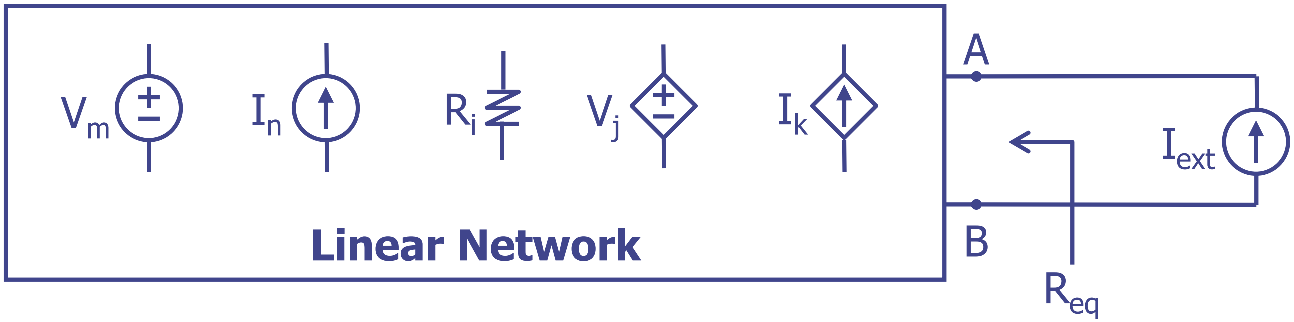
$$\Rightarrow \quad V_{ab} = -4I = 4 \text{ V}$$

$$\therefore \quad R_t = V_{ab}/1 \text{ A} = 4 \text{ V}/1 \text{ A} = 4\ \Omega$$

2.6.2 General Proof of Thevenin's Theorem

Consider a resistive linear network having M independent voltage sources, N independent current sources, and a number of dependent voltage and current sources.

Single out a port of the network and connect an external current source to it.



Then by superposition, we can find V_{AB} by summing the contributions of the independent sources taken one at a time including I_{ext}. All dependent sources must remain operative:

$$V_{AB} = \sum_{m=1}^{M} A_m V_m + \sum_{n=1}^{N} B_n I_n + I_{ext} R_{eq}$$

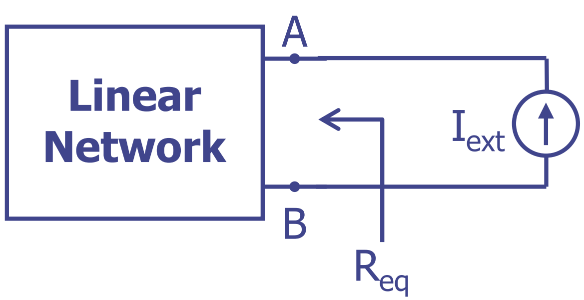
General Proof of Thevenin's Theorem (Cont.)

$$V_{AB} = \underbrace{\sum_{m=1}^{M} A_m V_m + \sum_{n=1}^{N} B_n I_n}_{V_{OC}} + I_{ext} R_{eq}$$

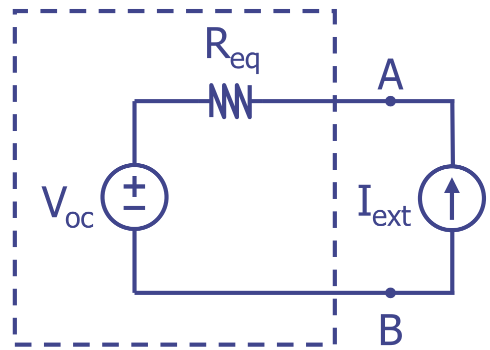
Notice that the first two terms constitute the open-circuit voltage, because this is the voltage when I_{ext} is set to zero. R_{eq} is the equivalent resistance looking into the port. So the I-V characteristic of the Linear Network is now reduced to

$$V_{AB} = V_{OC} + I_{ext} R_{eq}$$

But this is identical to the I-V characteristic of its Thevenin’s circuit shown on the right.



$\equiv$



This is true for all values of I_{ext} and V_{AB}. Hence the two circuits are equivalent.